Tôhoku Math. Journ.
21(1969), 328-335.

SOME REMARKS ON ANALYTIC CONTINUATIONS

SHINJI YAMASHITA

(Received January 31, 1969)

1. The purpose of the present paper is to prove some theorems concerning continuations of analytic functions across simple open arcs. Here, a simple open arc means a topological image of the open interval $\{t; 0 < t < 1\}$.

Let D_1 and D_2 be Jordan domains in the z-plane having no point in common and I be a simple open arc lying on the non-empty common boundary of D_1 and D_2. Then there arises the following

PROBLEM. Given two analytic functions f_1 and f_2 in D_1 and D_2 respectively, we set $f = f_1$ in D_1 and $f = f_2$ in D_2. Under what condition do there exist an open subset I^* of I and an analytic function $F(z)$ in $D_1 \cup I^* \cup D_2$ such that $F(z) = f_j(z)$ for $z \in D_j$ $(j = 1, 2)$? In other words, under what conditions on f and I can f be extended analytically to an open subset I^* of I?

This problem was investigated by some authors, e.g., Carleman [5], Wolf [14], Meier [8] and from cluster-sets-theoretic viewpoint, Bagemihl [3] gave an answer to this problem under the restriction of I being an open interval on a straight line. Recently, Noshiro [9] gave an improvement of Bagemihl's theorem [3] (cf. also [10]).

First in §2 we shall prove an analogous theorem to Bagemihl-Noshiro's in the case where I is an open locally rectifiable arc. Instead of the condition (c) in Theorem 6 in [9] we shall give a global restriction to f. In §3 we assume that I is a simple open smooth arc. We give an answer to the problem under the condition that f_j belongs to the Hardy class $H_p(D_j)$ for $p > 1$ $(j=1, 2)$. In §4 we assume that I is a simple open analytic arc. Under the weaker condition that f_j is in the class $H_1(D_j)$ $(j=1, 2)$, we shall give another answer to the problem. Finally in §5 we shall state some remarks on null-sets for the class H_p, $p \geqq 1$, as applications of two theorems in §3 and in §4.

2. By an open locally rectifiable arc I we mean a simple open arc such that every point of I has a neighbourhood which is a rectifiable subarc of I.