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ON THE EQUIVALENCE IN BANACH SPACES OF A CONVERGENCE THEOREM OF BERNSTEIN TYPE, A HAUSDORFF TYPE MOMENT PROBLEM AND A REGULARITY THEOREM

G. E. PETERSON AND DON H. TUCKER

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1. Introduction. Suppose each of X and Y is a linear normed space and $B=B[X,Y]$ is the space of bounded linear transformations from X into Y. Following the notation in [3] and [4], we denote the weak sequential extension of B by B^+ and of Y by Y^+. We shall denote by C the space of continuous functions on [0, 1] with values in X, the space having the uniform norm topology, and by C_0 the subspace of C consisting of those functions f such that $f(0)=\theta_X$, the zero element of X.

For each $\alpha \geqq 0$, C_α will denote C_0 if $\alpha > 0$ and it will signify C if $\alpha = 0$. We shall suppose throughout that $\Phi = \{\varphi_k\}_{k=0}^\infty$ represents a sequence of elements of B.

DEFINITION 1.

$$B_n^\alpha[\Phi, f] = \sum_{k=0}^{n} \binom{n+\alpha}{n-k} [\Delta^{n-k}\varphi_k] \cdot f\left(\frac{k}{n}\right)$$

will be called the (n, α)-Bernstein transform of f for each $f \in C$. Here we understand that

$$\binom{n+\alpha}{n-k} = \frac{\Gamma(n+\alpha+1)}{(n-k)!\,\Gamma(k+\alpha+1)} \qquad \text{for } 0 \leqq k < n,$$

$$= 1 \qquad \text{for } k = n,$$

$$= 0 \qquad \text{for } k > n,$$

and

$$\Delta^\nu \varphi_k = \sum_{m=0}^{\nu} (-1)^{\nu-m} \binom{\nu}{m} \varphi_{k+\nu-m} \qquad \text{for } \nu, k = 0, 1, 2, \cdots.$$