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SUMMABILITY IN TOPOLOGICAL GROUPS, IV (CONVERGENCE FIELDS)

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Introduction. In the papers [7], [8], and [9], I introduced the notions and notations of summability theory in the topological group setting. In [7] the groups were assumed Hausdorff and first countable. Some general theorems of the Toeplitz-Schur type were given. In [9] the fact of the existence of a right invariant metric was used and some theorems about the bounded convergence field of a regular limitation method were given. In this paper I shall investigate some (non-metric) properties of the convergence field of a method. The results generalize some theorems obtained by Agnew, Buck, Hill, Keough and Petersen, and the author. The reader may refer to their works listed in the references at the end of this paper. In all that follows G will denote a topological Hausdorff group which satisfies the first axiom of countability.

It is well-known that no regular matrix has all bounded sequences in its convergence field. For generalizations to topological groups see [7, page 265] and [9, Theorem 2, Corollary]. A related result asserts that for each regular matrix there is a strictly stronger matrix. This result generalizes as follows.

THEOREM 1. *Let G be a non-trivial group and let $\{f(m)\}$ be a regular triangular limitation method on G. Then there exists a regular triangular limitation method, $\{g(m)\}$, on G which is consistent with $\{f(m)\}$ and whose convergence field properly contains the convergence field of $\{f(m)\}$.*

PROOF. Let $\{f(m)\}$ be a regular triangular limitation method on G and let x be a non-zero element of G. Let $\{U(i)\}$ be a basic sequence of neighborhoods of 0 with $U(i+1)\subset U(i)$ for $i=1, 2,\cdots$. By condition (2) of the Toeplitz type theorem (Theorem 1, page 261, of [7]) there is a positive

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