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ON THE LOCAL BEHAVIOR OF FUNCTION ALGEBRAS

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1. In this paper we shall give complete proofs of theorems announced recently in [8]. These results are concerned with properties of interpolation sets and essential set with respect to a function algebra, and some of them are regarded as generalization of those results established in several literatures [4], [7] and [9]. Our main results are Theorem 1 and Theorem 3, which state that a w-interpolation set is not compatible with the essential set of the function algebra. By using these results, in some cases we can determine the essential set of the function algebra from those essential sets of the restriction algebras of countable closed partitions (Theorem 4). Theorem 3, together with Theorem 4, has been previously treated by Mullins [9] under the assumptions that the representing space X of the function algebra coincides with M_A and M_A is metrizable.

Let X be a compact Hausdorff space and $C(X)$ the algebra of all complex-valued continuous functions on X. We consider a function algebra A on X, that is, A is closed subalgebra of $C(X)$ which separates the points of X and contains the constants. In this case X is sometimes called a representing space of A. Throughout this paper M_A will indicate the maximal ideal space of A. The Šilov boundary of A will be denoted by ∂A. For a subset F in X, we shall denote by $A|F$ the restricted algebra of A to F. Let F be a closed subset of X. If $A|F$ is closed in $C(F)$, $A|F$ is regarded as a function algebra on F. Then F is called an interpolation set for A when $A|F=C(F)$. Let G be an open subset in X. G is called a w-interpolation set for A in X if any compact subset of G is an interpolation set for A in X. Following Bear [2] we define the essential set E of A in X as the set which is the hull of the largest ideal of $C(X)$ contained in A.

2. Bishop [3] and Glicksberg [6] have proved that a function algebra A is characterized by the restriction algebra on the disjoint partition of its maximal antisymmetric sets and Tomiyama [12] has shown that among these maximal antisymmetric sets the set P of points, each of which makes itself a maximal antisymmetric set, is free from the representing space X and plays a special rôle in determining the essential set of A; in fact