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APPROXIMATION OF SEMI-GROUPS OF NONLINEAR OPERATORS

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1. Introduction. Let us consider an evolution equation

$$(d/dt)u(t) = Au(t),\ u(0) = x \tag{1.1}$$

in a Banach space X. Here A is an operator, not necessarily linear, in X and is assumed to be time independent. And we introduce an approximating scheme to the evolution equation as follows. Take a sequence $\{h_n\}$ of positive numbers going to 0 as $n\to\infty$. The solution $u_n(t)$ to the n-th approximating equation is calculated inductively, for t integral multiples of h_n, by the following system of equations:

$$u_n((k+1)h_n) = C_n u_n(kh_n),\ u_n(0) = x \tag{1.2}$$

for $k = 0, 1, 2, \cdots$ and n, where each C_n is an operator from X into itself. In case when A is a linear operator whose domain $D(A)$ is dense in X, Trotter [13] proved the following results which show the existence of solution $u(t) = u(t;x)$ of (1.1) and the convergence of approximating solutions, that is,

$$u_n([t/h_n]h_n) = C_n{}^{[t/h_n]}x \to u(t\,;x),$$

where $[\cdot]$ denotes the Gaussian blacket.

THEOREM A. *Let $\{C_n\}$ be a sequence of bounded linear operators satisfying the consistency condition and the stability condition*:

$$\text{(C1)} \qquad \lim_{n\to\infty} h_n^{-1}(C_n - I)x = Ax \qquad \textit{for } x \in D(A)$$

and the domain $D(A)$ is dense linear in X,

$$\text{(S1)} \qquad \|C_n{}^k\| \leqq Ke^{Mkh_n} \qquad \textit{for } k \textit{ and } n,$$