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UNRAMIFIED EXTENSIONS OF QUADRATIC NUMBER FIELDS, II

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We have studied equations of type $X^n - aX + b = 0$, and have obtained some results on unramified extensions of quadratic number fields [3]. In this paper we have further results which include almost all of [3]. We do not refer to [3] in the following, though the techniques of proofs are almost equal to those of [3]. Theorems proved here are the following.[1] Notice that "unramified" means in this paper that every finite prime is unramified.

THEOREM 1. *Let k be an algebraic number field of finite degree. Let a and b be integers of k. K denotes the minimal splitting field of a polynomial*

$$f(X) = X^n - aX + b,$$

i.e., $K = k(\alpha_1, \cdots, \alpha_n)$ where $\alpha_1, \cdots, \alpha_n$ are the roots of $f(X) = 0$. Let $D = \prod_{i<j} (\alpha_i - \alpha_j)^2$ be the discriminant of $f(X)$. If $(n-1)a$ and nb are relatively prime, K is unramified over $k(\sqrt{D})$.

THEOREM 2. *Let $n \geqq 3$ be an integer, and A_n be an alternating group of degree n. Then there exist infinitely many quadratic number fields which have unramified Galois extensions with Galois groups A_n.*

1. Proof of Theorem 1. Let $\mathfrak{P}$ be any finite prime of K, and let $\mathfrak{p} = \mathfrak{P} \cap k$. Let G be the Galois group of K over k. Then G is a permutation group of $(\alpha_1, \cdots, \alpha_n)$. Let H be the subgroup of G consisting of the even permutations. H corresponds to $k(\sqrt{D})$. We shall prove Theorem 1 by showing that H meets with the inertia group of $\mathfrak{P}$ trivially. First we consider the factorization of $f(X)$ mod $\mathfrak{p}$. From $f(X) = X^n - aX + b$ and $f'(X) = nX^{n-1} - a$, it follows

$$Xf'(X) - nf(X) = (n-1)aX - nb.$$

1) After I prepared the manuscript of this paper, I knew that Y. Yamamoto had already obtained the same results which is to appear in Osaka Math. J. before long.