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A COMPLEMENT TO "ON THE HOMOMORPHISM OF VON NEUMANN ALGEBRA"

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In the proof of Theorem I in [1], the continuum hypothesis has been used. Dr. J.Vesterstrφm has asked the author whether Theorem I in [1] can be proved or not without using the continuum hypothesis. In this paper, we shall show that the fact holds without the continuum hypothesis. Before going into the discussions, the author wishes to his thanks to Dr. J. Vesterstrφm for his indebted communication in the presentation of this paper.

THEOREM. *Let M be a properly infinite von Neumann algebra with the separable predual M_*; then any homomorphism π from M onto a von Neumann algebra N is σ-weakly continuous.*

PROOF. In [1], we have shown the following fact, that is, if any *-homomorphism from a von Neumann algebra M onto a von Neumann algebra N is σ-weakly continuous, then any homomorphism from M onto N is σ-weakly continuous. Therefore, to prove Theorem, we may suppose that π is a *-homomorphism.

If the kernel $\pi^{-1}(0)=I$ of π is σ-weakly closed, then there exists a central projection z of M such that $I=M_{(1-z)}$. Then, the restriction of π to Mz is a *-isomorphism from Mz onto N. Therefore, π is σ-weakly continuous. Next, we assume that I is not σ-weakly closed and derive a contradiction. Let $\tilde{I}$ be the σ-weak closure of I in M. Then there exists a central projection z of M such that $\tilde{I}=Mz$. Since π is a *-homomorphism from M onto N, $\pi(z)$ is a central projection of N. Furthermore, π induces a *-homomorphism from Mz onto $N_{\pi(z)}$. Therefore, we can assume that π is a *-homomorphism from M onto N and the kernel $\pi^{-1}(0)=I$ is σ-weakly dense in M. Since the predual M_* is separable, M is σ-finite and the cardinal number of M_p is not larger than $\mathfrak{c}$.

Now, since the σ-weak closure of I is M and M is σ-finite, there exists a sequence $\{e_n\}_{n=1}^{\infty}$ of mutually orthogonal projections in I with $\sum_{n=1}^{\infty} e_n=1$. That is, there exists a family $\{e_n\}_{n=1}^{\infty}$ of countable orthogonal projections such that $\sum_{n=1}^{\infty} e_n$