Tôhoku Math. Journ.
22(1970), 174-180.

SOME REMARKS ON LOCAL MARTINGALES

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(Received April 9, 1969)

Let (Ω, F, P) be a basic probability space where F is complete with respect to P and let $\{F(t)\}_{0\leqq t<\infty}$ be an increasing family of Borel subfields of F. In what follows, we suppose that the family $\{F(t)\}_{0\leqq t<\infty}$ is right continuous and has no time of discontinuity.

We call a family $T=\{F(t), \tau_t\}_{0\leqq t<\infty}$ a *time change function* with respect to the family $\{F(t)\}_{0\leqq t<\infty}$, if

(1) for each $u\in[0, \infty)$, τ_u is a stopping time with respect to the family $\{F(t)\}_{0\leqq t<\infty}$ and $\tau_u<\infty$,

(2) for almost all ω, $[0, \infty)\ni u\to\tau_u(\omega)$ is a continuous and strictly increasing function with $\tau_0(\omega)=0$.

For a right continuous stochastic process $X=\{x_t, F(t)\}_{0\leqq t<\infty}$ and a time change function $T=\{F(t), \tau_t\}_{0\leqq t<\infty}$, we can define a new stochastic process $TX=\{x_{\tau_t}, F(\tau_t)\}_{0\leqq t<\infty}$ and we call it the stochastic process obtained from X by a time change with respect to T. In particular, if $X^a=\{x_t^a, F(t)\}_{0\leqq t<\infty}$, $a\in A$, where A is an arbitrary set, is a collection of continuous stochastic processes such that

$$\sup\{|x_s^a-x_0^a|;\ 0\leqq s\leqq t,\ a\in A\}$$

is continuous, then we call the time change function

$$\Theta=\{F(t), \theta_t\}_{0\leqq t<\infty},$$

the *stopping process* or the *brake* of the processes X^a, where $\theta_t=\inf\{u;\ \lambda_u>t\}$ and $\lambda_t=t+\sup\{|x_s^a-x_0^a|;\ 0\leqq s\leqq t,\ a\in A\}$.

In the followings we assume that $x_0=0$ and X is quasi-continuous from the left. We call a martingale $X=\{x_t, F(t)\}_{0\leqq t<\infty}$ an *L_∞^*-martingale* if for each t

$$P\{\sup_{0\leqq u\leqq t}|x_u|\leqq c_t\}=1$$

where c_t is some constant with $c_0=c_{0+}=0$. Let M designate the set of all right continuous local martingales which can be transformed into L_∞^*-martingales