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# A NOTE ON A THEOREM OF E. CARTAN

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In G. Harder [5] he has proved a theorem of E. Cartan. In this paper we give another proof of the following main theorems in his paper, using the terminology of Galois cohomology.

THEOREM 1. *Let $G$ be a connected algebraic group defined over the real number field $R$, and of compact form, $T$ a maximal torus of $G$, defined over $R$. Then the kernel of the canonical mapping $H^1(R,T)\to H^1(R,G)$ consists of unit element, i.e.,* $\operatorname{Ker}\{H^1(R,T)\to H^1(R,G)\}=1$.

REMARK 1. On the above theorem in Harder's paper, he has assumed that $G$ is semi-simple, but we can avoid this condition. Moreover he has concluded that $H^1(R,T)\to H^1(R,G)$ is injective in the sense of the category of point sets, therefore it is necessary to explain that this mapping is injective in the set theoretical sense. But in the Remark of §2 in this paper, we shall point out his error by using this terminology.

REMARK 2. Let $G$ be an algebraic group defined over a field $k$. Then $G$ is called $k$-compact or $k$-anisotropic, if $G$ has no non-trivial $k$-Borel subgroup. When $G$ is reductive, $G$ is $k$-compact if and only if $G$ has no non-trivial $k$-split torus. If $k$ is a local field, say $R$ or $Q_p$ ($p$-adic field), then $G$ is $k$-compact if and only if the group $G_k$, consisting of $k$-rational elements of $G$, is compact.

THEOREM 2. (E. Cartan's theorem). *Let $G$ be a semi-simple algebraic group defined over $R$. Then the maximal tori defined over $R$ of $G$ are conjugate over $R$ each other.*

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**1.** Let $G$ be an algebraic group defined over $R$. Then $H^1(R,G_R)$ is the set of classes on $G_R$ modulo conjugate of elements such that $x^2=1$, where $G_R$ consists of $R$-rational elements of $G$. Therefore if $H$ is a subgroup of $G$ defined over $R$,