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THE EQUATION $\Delta u = Pu$ ON E^m WITH ALMOST ROTATION FREE $P \geqq 0$

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Consider a connected C^∞ Riemannian m-manifold R $(m \geqq 2)$ and a continuously differentiable function P ($\geqq 0$ and $\not\equiv 0$) on R. The space of solutions of $d{*}du = Pu{*}1$ or $\Delta u = Pu$ on R will be denoted by $P(R)$. Let $\mathcal{O}_{PX}$ be the set of pairs (R, P) such that the subspace $PX(R)$ of $P(R)$ consisting of functions with a certain property X reduces to $\{0\}$. Here we let X be B which stands for boundedness, D for the finiteness of the Dirichlet integral $D_R(u) = \int_R du \wedge {*}du$, and E for the finiteness of the energy integral $E_R^P(u) = D_R(u) + \int_R Pu^2{*}1$; we also consider nontrivial combinations of these properties. We denote by $\mathcal{O}_G$ the set of pairs (R, P) such that there exists no harmonic Green's function on R.

The purpose of this paper is to show that (E^m, P) will be an example for the strictness of each of the following inclusion relations

$$(1) \qquad \mathcal{O}_G \subset \mathcal{O}_{PB} \subset \mathcal{O}_{PD} \subset \mathcal{O}_{PE}$$

if P is properly chosen, where $E^m (m \geqq 3)$ is m-dimensional Euclidean space and P is a continuously differentiable function on $E^m (\geqq 0, \not\equiv 0)$.

More precisely let

$$(2) \qquad P(x) \sim |x|^{-\alpha}$$

as $|x| \to \infty$, i.e. there exists a constant $c > 1$ such that $c^{-1}|x|^{-\alpha} \leqq P(x) \leqq c|x|^{-\alpha}$ for large $|x|$. Then the following is true:

$$(3) \qquad \begin{cases} (E^m, P) \in \mathcal{O}_{PB} - \mathcal{O}_G \text{ if } \alpha \leqq 2; \\ (E^m, P) \in \mathcal{O}_{PD} - \mathcal{O}_{PB} \text{ if } 2 < \alpha \leqq (m+2)/2; \\ (E^m, P) \in \mathcal{O}_{PE} - \mathcal{O}_{PD} \text{ if } (m+2)/2 < \alpha \leqq m. \end{cases}$$

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