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MINIMAL SUBMANIFOLDS WITH M-INDEX 2 IN RIEMANNIAN MANIFOLDS OF CONSTANT CURVATURE

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For a submanifold M in a Riemannian manifold $\bar{M}$, the minimal index (M-index) at a point of M is defined by the dimension of the linear space of all 2nd fundamental forms with vanishing trace. The geodesic codimension of M in $\bar{M}$ is defined by the minimum of codimensions of M in totally geodesic submanifolds of $\bar{M}$ containing M.

It is clear in general that for M in $\bar{M}$

$$M\text{-index} \leqq \text{geodesic codimension}.$$

In [7], the author investigated minimal submanifolds with M-index 2 in Riemannian manifolds of constant curvature and gave some typical examples of such submanifolds with geodesic codimension 3 in the space forms which is quite analogous to the case of helicoids in E^3 when $\bar{M}$ is Euclidean. In the present paper, he will give some examples of such submanifolds with geodesic codimension 4 in the space forms. In the previous case, the base surface (analogous to the helix for a helicoid) must be locally flat, but in the present case it must be of positive constant curvature.

We will use the notations in [7].

1. Preliminaries. Let $\bar{M}=\bar{M}^{n+\nu}$ be a Riemannian manifold of dimension $n+\nu$ and of constant curvature $\bar{c}$ and $M=M^n$ be an n-dimensional submanifold in $\bar{M}$. Let $\bar{\omega}_A$, $\bar{\omega}_{AB}=-\bar{\omega}_{BA}$, $A,B=1,2,\cdots,n+\nu$, be the basic and connection forms of $\bar{M}$ on the orthonormal frame bundle $F(\bar{M})$ which satisfy the structure equations

$$(1.1)\qquad d\bar{\omega}_A=\sum_B \bar{\omega}_{AB}\wedge{}_B\bar{\omega},\ d\bar{\omega}_{AB}=\sum_C \bar{\omega}_{AC}\wedge\bar{\omega}_{CB}-\bar{c}\,\bar{\omega}_A\wedge\bar{\omega}_B\,.$$

Let B be the subbundle of $F(\bar{M})$ over M such that $b=(x,e_1,\cdots,e_n,\cdots,e_{n+\nu})\in F(\bar{M})$ and $(x,e_1,\cdots,e_n)\in F(M)$, where $F(M)$ is the orthonormal frame bundle of M with the induced Riemannian metric from $\bar{M}$, then deleting the bars of $\bar{\omega}_A$, $\bar{\omega}_{AB}$ on B, we have

$$(1.2)\qquad \omega_\alpha=0,\ \omega_{i\alpha}=\sum_j A_{\alpha ij}\omega_j,\ A_{\alpha ij}=A_{\alpha ji}\quad \alpha=n+1,\cdots,n+\nu;\ i,j=1,2,\cdots,n.$$