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ON POSITIVE CONVOLUTION OPERATORS FOR JACOBI SERIES

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1. Introduction.

1.1. In a preceding paper [2] the author has started the study of approximation of functions by processes, which are generated by the use of summability methods for the expansion of the functions in terms of Jacobi polynomials. The summability methods can be interpreted as convolution operators, if the convolution structure for Jacobi series, defined by Askey and Wainger [1], is used. By means of some general theorems on approximation processes on Banach spaces, (Berens [3]), it is possible to characterize the saturation class and the classes of non-optimal approximation of a number of classical summability methods for the summation of the Fourier-Jacobi series. This paper deals with saturation of positive convolution operators and the main part is a theorem of the Tureckii [10]—DeVore [4] type, which determines the saturation order and the saturation class of a sequence of positive convolution operators, satisfying a special condition on the Fourier-Jacobi coefficients of the kernel. The proof is a straight-forward generalization of DeVore's proof in the case of Fourier series. As applications, the saturation class of the higher order Jackson kernel and some other positive kernels are characterized.

1.2. We introduce some Banach spaces of complex valued functions on the interval $[-1, 1]$. We write C for the space of continuous functions, L^∞ denotes the space of essentially bounded functions and we define the L^p spaces with respect to the weight function ($x = \cos\theta$)

$$\rho^{(\alpha,\beta)}(\theta) = \left(\sin\frac{\theta}{2}\right)^{2\alpha+1}\left(\cos\frac{\theta}{2}\right)^{2\beta+1} \qquad \left(\alpha \geq \beta \geq -\frac{1}{2}\right). \tag{1.1}$$

We call M the space of all regular finite Borel measures on $[-1, 1]$. The spaces C, $L^p (1 \leq p \leq \infty)$ and M are Banach spaces if endowed with the following norms