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SOME UNIQUENESS THEOREMS FOR $H^p(U^n)$ FUNCTIONS

Dedicated to Professor Gen-ichirô Sunouchi on his 60th birthday

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1. Let U^n denote the unit polydisc $\{(z_1, \cdots, z_n); |z_1| < 1, \cdots, |z_n| < 1\}$ in the n-dimensional complex vector space C^n, T^n the distinguished boundary of U^n and m_n the normalized Haar measure on T^n. A function $f(z)$, holomorphic in U^n, is said to be of class $H^p(U^n)$, $N(U^n)$ or $N_*(U^n)$, if

$$\|f\|_p = \sup_{0<r<1}\left(\int_{T^n}|f(rw)|^p dm_n(w)\right)^{1/p} < \infty ,$$

$\sup_{0<r<1}\int_{T^n}\log^+|f(rw)|dm_n(w) < \infty$ or $\{\log^+|f(rw)|, 0 < r < 1\}$ forms a uniformly integrable family on T^n respectively. It is known (see Rudin [6]) that if $f \in N(U^n)$, $\lim_{r\to 1} f(rw) = f^*(w)$ exists for almost all $w \in T^n$ and $\log|f^*| \in L^1(T^n)$ when $f \not\equiv 0$, and we have $N(U^n) \supset N_*(U^n) \supset H^p(U^n) \supset H^q(U^n)$ (if $0 < p < q \leqq \infty$). It is also known that an $f \in N_*(U^n)$ is of class $H^p(U^n)$ if and only if $f^* \in L^p(T^n)$. A function $f(z)$ is said to be outer if $f \in N_*(U^n)$ and $\log|f(0)| = \int_{T^n}\log|f^*(w)|dm_n(w)$. It can be shown easily that an f is outer if and only if f and $1/f \in N_*(U^n)$. This follows from the fact that an $f \in N(U^n)$ lies in $N_*(U^n)$ if and only if

$$\log|f(z)| \leqq \int P(z, w)\log|f^*(w)|dm_n(w) \qquad \text{in } U^n ,$$

where $P(z, w)$ is the Poisson kernel for U^n, i.e. $P(z, w) = \prod_{j=1}^n (1-r_j^2)/(1 - 2r_j\cos(\theta_j - \varphi_j) + r_j^2)$ if $z_j = r_j e^{i\theta_j}$ and $w_j = e^{i\varphi_j}$, (Rudin [6], p. 47).

In this note we shall show three uniqueness theorems for $H^1(U^n)$ functions of which only the arguments on the boundary are given. Relating to these, we shall give geometric aspects of outer functions in section 2 and some uniqueness theorems for other classes of holomorphic functions in U^n in section 3. Some applications are given in section 5.

The main results are the followings.

THEOREM 1. [8]. *Let $f \in H^1(U^n)$, be outer and $1/f^* \in L^p(T^n)$ $(1/2 \leqq p \leqq 1)$.*

We use systematically the notations in Rudin [6].