

ON THE GENERATION OF SEMI-GROUPS OF LINEAR OPERATORS

Dedicated to Professor Gen-ichirô Sunouchi on his 60th birthday

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1. Introduction. This paper is concerned with the generation of semi-groups of classes $(0, A)$ and $(1, A)$.

Let X be a Banach space and let $B(X)$ be the set of all bounded linear operators from X into itself. A one-parameter family $\{T(t); t \geq 0\}$ is called a *semi-group* (of operators), if it satisfies the following conditions:

$$(1.1) \quad T(t) \in B(X) \text{ for } t \geq 0.$$

$$(1.2) \quad T(0) = I \text{ (the identity), } T(t+s) = T(t)T(s) \text{ for } t, s \geq 0.$$

$$(1.3) \quad \lim_{h \rightarrow 0} T(t+h)x = T(t)x \text{ for } t > 0 \text{ and } x \in X.$$

Let $\{T(t); t \geq 0\}$ be a semi-group. By the *infinitesimal generator* A_0 of $\{T(t); t \geq 0\}$ we mean

$$(1.4) \quad A_0 x = \lim_{h \rightarrow 0+} (T(h)x - x)/h$$

whenever the limit exists. If A_0 is closable, then $A = \bar{A}_0$ (the closure of A_0) is called the *complete infinitesimal generator* of $\{T(t); t \geq 0\}$.

The following basic classes of semi-groups are well known (see [2]). If a semi-group $\{T(t); t \geq 0\}$ satisfies the condition $(C_0) \lim_{t \rightarrow 0+} T(t)x = x$ for $x \in X$, then $\{T(t); t \geq 0\}$ is said to be of *class* (C_0) . In this case A_0 is closed and hence the complete infinitesimal generator coincides with the infinitesimal generator. If a semi-group $\{T(t); t \geq 0\}$ satisfies the condition

$$(1, A) \quad \int_0^1 \|T(t)\| dt < \infty \text{ and } \lim_{\lambda \rightarrow \infty} \lambda \int_0^\infty e^{-\lambda t} T(t)x dt = x \text{ for } x \in X,$$

then $\{T(t); t \geq 0\}$ is said to be of *class* $(1, A)$. If, instead of the condition $(1, A)$, $T(t)$ satisfies the weaker condition

$$(0, A) \quad \int_0^1 \|T(t)x\| dt < \infty \text{ and } \lim_{\lambda \rightarrow \infty} \lambda \int_0^\infty e^{-\lambda t} T(t)x dt = x \text{ for } x \in X,$$

then a semi-group $\{T(t); t \geq 0\}$ is said to be of *class* $(0, A)$. Clearly $(C_0) \subset (1, A) \subset (0, A)$ in the set theoretical sense. It is known that in general the infinitesimal generator of a semi-group of class $(1, A)$ need not