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TAUBERIAN THEOREMS FOR $(\mathfrak{R}, p, \alpha)$ SUMMABILITY

Dedicated to Professor Gen-ichirô Sunouchi on his 60th birthday

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1. Introduction. Professor G. Sunouchi has introduced the summability $(\mathfrak{R}, \alpha)$ and $(\mathfrak{R}^*, \alpha)$ in his paper [4]. Later we [3] have introduced, as generalizations of these summability, the summability $(\mathfrak{R}, p, \alpha)$ defined as follows. Throughout this paper, p denotes a positive integer and α denotes a real number, not necessarily an integer, such that $0 < \alpha < p$. Let us put

$$C_{p,\alpha} = \int_0^\infty \frac{\sin^p x}{x^{\alpha+1}}\,dx\ ,$$

$$\varphi(n, t) \equiv \varphi(nt) \equiv (C_{p,\alpha})^{-1}\int_{nt}^\infty \frac{\sin^p x}{x^{\alpha+1}}\,dx = (C_{p,\alpha})^{-1}\int_t^\infty \frac{\sin^p nu}{n^\alpha u^{\alpha+1}}\,du\ . \tag{1.1}$$

Then a series $\sum_{n=0}^\infty a_n$ is said to be summable $(\mathfrak{R}, p, \alpha)$ to s if the series in

$$f(p, \alpha, t) = a_0 + \sum_{n=1}^\infty a_n\varphi(nt)$$

converges for t positive and small and $f(p, \alpha, t) \to s$ as $t \to +0$. Under this definition, the summability $(\mathfrak{R}, \alpha)$ and the summability $(\mathfrak{R}^*, \alpha)$ are reduced to the summability $(\mathfrak{R}, 1, \alpha)$ and the summability $(\mathfrak{R}, 2, \alpha)$, respectively. On the other hand, for a series $\sum a_n$, let us write $\sigma_n^\beta = s_n^\beta/A_n^\beta$, where s_n^β and A_n^β are defined by the relations

$$(1-x)^{-\beta-1} = \sum_{n=0}^\infty A_n^\beta x^n \quad\text{and}\quad (1-x)^{-\beta-1}\sum_{n=0}^\infty a_n x^n = \sum_{n=0}^\infty s_n^\beta x^n\ . \tag{1.2}$$

Then, if $\sigma_n^\beta \to s$ as $n \to \infty$, we say that the series $\sum_{n=0}^\infty a_n$ is summable (C, β), $\beta > -1$, to s.

Concerning $(\mathfrak{R}, p, \alpha)$ summability, we [3] have proved the following theorems.

THEOREM A. *Let $0 < \beta < \alpha < p$. Then, if a series $\sum_{n=0}^\infty a_n$ is summable (C, β) to s, the series $\sum_{n=0}^\infty a_n$ is summable $(\mathfrak{R}, p, \alpha)$ to s.*

THEOREM B. *Let $0 < \alpha < p$, $\lambda_n > 0$ $(n = 1, 2, 3, \cdots)$ and the series $\sum_{n=1}^\infty \lambda_n/n$ converge. Then, if*