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LOCAL PROPERTY OF THE SINGULAR SETS OF SOME KLEINIAN GROUPS

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Introduction. In the recent paper [3], I proved the existence of Kleinian groups with fundamental domains bounded by four circles whose singular sets have positive 1-dimensional measure. Now in the natural way the following problem arises; to what extent does the Hausdorff dimension of the singular sets of Kleinian groups climb up, when the number N of the boundary circles increases? It is conjectured and seems still open that the 2-dimensional measure of the singular sets E of general finitely generated Kleinian groups is always zero (see [1]).

The purpose of this paper is to investigate the properties of computing functions introduced in No. 3 of §1 in detail and the local property of the singular set of the Kleinian group by using these properties.

We shall state preliminaries and notations about Kleinian groups in §1. We shall prove the main theorem giving the relation between the computing function and the Hausdorff measure of the singular set of Kleinian group in §2. At last in §3 we shall seek for the relation between the computing function and the Hausdorff dimension of the singular set and further give an application to the convergence problem of Poincaré theta-series by using the main theorem.

§1. Preliminaries and Notations.

1. Let us denote by B the domain bounded by N mutually disjoint circles H_i, $H_i'(1 \leqq i \leqq p)$ and $K_j(1 \leqq j \leqq q)$ and form the properly discontinuous group of linear transformations with the fundamental domain B, where $N = 2p + q$.

Let S_i be a hyperbolic or loxodromic generator which transforms the outside of H_i onto the inside of H_i'. Then $\{S_i\}_{i=1}^p$ generates a Schottky group whose fundamental domain is bounded by $\{H_i, H_i'\}_{i=1}^p$. Let $\{S_j^*\}_{j=1}^q$ be the elliptic transformations with period 2 corresponding to $\{K_j\}_{j=1}^q$. Then $\{S_j^*\}_{j=1}^q$ generates a properly discontinuous group whose fundamental domain is the outside of the boundary circles $\{K_j\}_{j=1}^q$.

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