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NORMALITY OF ALMOST CONTACT 3-STRUCTURE

Dedicated to Professor Shigeo Sasaki on his 60th birthday

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0. Introduction. The almost contact 3-structure has been defined by Kuo [5, 6], Tachibana [6, 12], Yu [12] and studied by them and Eum [16], Kashiwada [4], Ki [16], Sasaki [10], Yano [16]. Some topics related to almost contact 3-structures have been considered by Ishihara, Konishi [1, 2, 3] and Tanno [13].

It is well known that the product of a manifold with almost contact 3-structure and a straight line admits an almost quaternion structure (cf. [5]). Recently, Ako and one of the present authors [14, 15] have proved that, if for an almost quaternion structure (F, G, H) the Nijenhuis tensors $[F, F]$ and $[G, G]$ vanish, then the other Nijenhuis tensors $[H, H]$, $[G, H]$, $[H, F]$ and $[F, G]$ vanish too (cf. Obata [7]), and that if the Nijenhuis tensor $[F, G]$ vanishes, then the other Nijenhuis tensors $[F, F]$, $[G, G]$, $[H, H]$, $[G, H]$, and $[H, F]$ vanish too. The main purpose of the present paper is to study almost contact 3-structures in the light of this work.

1. Almost contact 3-structure. Let M be an n-dimensional differentiable manifold[1] and let f, U and u be a tensor field of type (1, 1), a vector field and a 1-form in M, respectively. If f, U and u satisfy

$$f^2 = -I + u \otimes U\,, \quad fU = 0\,, \quad u \circ f = 0\,, \quad u(U) = 1\,,$$

the 1-form $u \circ f$ being defined by $(u \circ f)(x) = u(fx)$[2] and I being the identity tensor field of type (1, 1), then the set (f, U, u) is called an *almost contact structure* (cf. [8, 9, 11]).

Let f_1, f_2 be tensor fields of type (1, 1), U_1, U_2 vector fields and u_1, u_2 1-forms in M. If (f_1, U_1, u_1) and (f_2, U_2, u_2) are both almost contact structures and satisfy

$$f_1 f_2 + f_2 f_1 = u_1 \otimes U_2 + u_2 \otimes U_1\,, \quad f_1 U_2 + f_2 U_1 = 0\,,$$
$$u_1 \circ f_2 + u_2 \circ f_1 = 0\,, \quad u_1(U_2) = 0\,, \quad u_2(U_1) = 0\,,$$

1) Manifolds, vector fields, tensor fields and other geometric objects we discuss are assumed to be differentiable and of class C^∞.

2) Here and in the sequel, x, y and z denote arbitrary vector fields in the manifold M.