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LIE ALGEBRAS IN WHICH EVERY FINITELY GENERATED SUBALGEBRA IS A SUBIDEAL

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1. Introduction.

1.1. We prove that: *To every positive integer n there exist positive integers $\lambda_1(n)$ and $\lambda_2(n)$ such that every Lie algebra all of whose $\lambda_2(n)$-generator subalgebras are n-step subideals is nilpotent of class $\leqq \lambda_1(n)$.*

This result is the Lie theoretic analogue of that by Roseblade [4]. We leave unanswered the question of whether or not we can replace $\lambda_2(n)$ by n. However we give an example which shows that if $\lambda_2(n)$ is replaced by $n-2$, then the result is false.

1.2. Notation. All Lie algebras considered in this paper (unless otherwise specified) will have finite or infinite dimension over a fixed (but arbitrary) field $\boldsymbol{k}$.

We employ the notation of [3] and [5].

Let L be a Lie algebra and H a subspace of L. By $H \leqq L$, $H \lhd L$, H si L, $H \lhd^m L$ we shall mean (respectively) that H is a subalgebra, an ideal, *subideal* (in the sense of Hartley [3] p. 257), and m-step subideal of L.

Square brackets $[\,,\,]$ will denote Lie multiplication and triangular brackets $\langle\,,\,\rangle$ will denote the subalgebra generated by their contents. If A, B are subsets of L, then $[A, B]$ is the subspace spanned by all $[a, b]$ with $a \in A$, $b \in B$; and inductively, $[A, {}_0B] = A$ and $[A, {}_nB] = [[A, {}_{n-1}B], B] (n > 0)$. We let $\langle A^B \rangle$ be the smallest subalgebra of L containing A and invariant under Lie multiplication by the elements of B. If A, B are subspaces we define $A \circ B = \langle [A, B]^C \rangle$, where $C = \langle A, B \rangle$; and inductively $A \circ_1 B = A \circ B$, $A \circ_{n+1} B = (A \circ_n B) \circ B$; and $A + B$ is the vector space spanned by A and B.

$L^{(n)}$, L^n, $Z_n(L)$ denote respectively the n-th terms of the derived series, lower central series and upper central series of L. Inductively we define $L^{(0)} = L$, $L^{(n)} = [L^{(n-1)}, L^{(n-1)}]$, $L^1 = L$, $L^{n+1} = [L^n, L]$, $Z_0(L) = 0$, $Z_n(L)/Z_{n-1}(L) = Z(L/Z_{n-1}(L)) (n > 0)$ where $Z(L) =$ centre of $L = \{x \in L \mid [x, L] = 0\}$.

If $H \leqq L$, then the ideal closure series of H in L,

$$\cdots H_i \lhd H_{i-1} \lhd \cdots \lhd H_0 = L\ ,$$