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ON AUTOMORPHISM GROUPS OF II_1-FACTORS

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1. Introduction. In the present paper, we shall study groups of *-automorphisms on II_1-factors, using various topologies.

One of the main purposes is to attack the problem whether every II_1-factor has outer *-automorphisms. In [7], we proved that the symmetry on $M \overline{\otimes} M$ is outer for every non-atomic factor M; therefore, it is very plausible that every II_1-factor may have outer *-automorphisms.

Now let M be a II_1-factor with the separable predual. Let $C(M)$ (resp. $H(M)$ and $T(M)$) be the set of all central (resp. hyper-central and trivial-central) sequences in M. If $H(M) \subsetneqq C(M)$, then by McDuff's theorem [2], M is *-isomorphic to $M \overline{\otimes} U$, where U is the hyperfinite II_1-factor, so that M has outer *-automorphisms.

Among other things, we shall show that if $T(M) \subsetneqq C(M)$, then M has outer *-automorphisms (Corollary 7).

2. Theorems. Let M be a W^*-algebra, and let $A^*(M)$ be the group of all *-automorphisms on M. Let $B(M)$ be the Banach algebra of all bounded linear operators on M and let M_* be the predual of M. By the standard theory of Banach spaces, $B(M)$ is the dual Banach space of $M \otimes_\gamma M_*$, where γ is the greatest cross norm. We shall consider the topology $\sigma(B(M), M \otimes_\gamma M_*)$ on $A^*(M)$. We call this topology on $A^*(M)$ the weak *-topology and denote it by w^*.

PROPOSITION 1. *Suppose that a directed set* (ρ_α) *of elements in* $A^*(M)$ *converges to* $\rho_0 \in A^*(M)$ *in the* w^**-topology; then for any* $a \in M$, $(\rho_\alpha(a))$ *converges to* $\rho_0(a)$ *in the* $s(M, M_*)$*-topology.*

PROOF. Let M_* be the set of all normal positive linear functionals on M; then for $\varphi \in M_*$,

$$\varphi((\rho_\alpha(a) - \rho_0(a))^*(\rho_\alpha(a) - \rho_0(a))) = \varphi(\rho_\alpha(a^*a) + \rho_0(a^*a) - \rho_\alpha(a^*)\rho_0(a) \\ - \rho_0(a^*)\rho_\alpha(a)) \rightarrow \varphi(\rho_0(a^*a) + \rho_0(a^*a) - \rho_0(a^*)\rho_0(a) - \rho_0(a^*)\rho_0(a)) = 0 .$$

Similarly,

$$\varphi((\rho_\alpha(a) - \rho_0(a))(\rho_\alpha(a) - \rho_0(a))^*) \rightarrow 0 .$$

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