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ON THE EMBEDDING AS A DOUBLE COMMUTATOR IN A TYPE 1 AW^*-ALGEBRA II

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One of the interesting problems in operator algebras is to find the essential condition for a C^*-algebra to have a faithful von Neumann algebra representation. In 1951, I. Kaplansky [4] introduced a class of C^*-algebras called AW^*-algebras to separate the discussion of the internal structure of a von Neumann algebra from the action of its elements on a Hilbert space and showed that much of the "non spatial theory" of von Neumann algebras can be extended to AW^*-algebras; in particular, the lattice structure of the set of projections, type classifications for algebras, etc. can be carried over to AW^*-algebras. However, as J. Dixmier [2] showed, this class of AW^*-algebras is exactly broader than that of von Neumann algebras.

In 1956, J. Feldman [3] showed that a finite AW^*-algebra with a separating set of states which are completely additive on projections can be represented faithfully as a von Neumann algebra and conjectured that the theorem is true without the assumption of finiteness. In 1970, the author [8] proved the same theorem for semi-finite case using the Segal's non-commutative integration theory, although its proof is rather long. Recently, G. K. Pedersen showed that any C^*-algebra with weakly closed maximal abelian *-subalgebra is necessarily a von Neumann algebra and as a consequence, he solved the Feldman's conjecture for the general case in the affirmative.

In this paper, the author will prove the following theorem which is the module setting of the Pedersen's idea on one side and is the complete solution of problem of embedding as a double commutator in a type 1 AW^*-algebra ([9, 10]) on the other side.

THEOREM 1. *Let B be an AW^*-algebra of type* 1 *with center Z and let A be an AW^*-subalgebra* (*the definition below*) *of B which contains Z, then the double commutator A'' in B* (*that is $A'' = \{x, x \in B, xy = yx$ for all $y \in A\}$*) *is, exactly, A.*

Main tools in this proof are H. Widom's double commutation theorem

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