Tôhoku Math. Journ.
27 (1975), 13-24.

ON THE CHARACTERISTIC FUNCTION OF HARMONIC KÄHLERIAN SPACES

YOSHIYUKI WATANABE

(Received July 25, 1973)

1. Introduction. An n-dimensional space of constant curvature ($k \neq 0$) is characterized as a harmonic Riemannian space with characteristic function

$$f(\Omega) = 1 + (n-1)\sqrt{2k\Omega}\cot\sqrt{2k\Omega}$$

where $\Omega = s^2/2$ and s means the geodesic distance. A. Lichnérowicz has obtained the following

THEOREM A. ([3], [10]) *In any harmonic Riemannian space H^n with positive definite metric, its characteristic function $f(\Omega)$ satisfies the inequality*

$$\dot{f}^2(0) + \frac{5}{2}(n-1)\ddot{f}(0) \leqq 0 . \tag{1.1}$$

The equality sign is valid if and only if H^n is of constant curvature.

Recently, S. Tachibana [7] has showed that a $2m$-dimensional space of constant holomorphic curvature ($k \neq 0$) is characterized as a harmonic Kählerian space with characteristic function given by

$$f(\Omega) = 1 + (2m-1)(ls)\cot(ls) - (ls)\tan(ls) , \tag{1.2}$$

or

$$f(\Omega) = 1 + (2m-1)(ls)\coth(ls) + (ls)\tanh(ls) \tag{1.2$'$}$$

according to $l = \sqrt{k}/2$, or $l = \sqrt{-k}/2$. He also has obtained

THEOREM B. *In any $n(=2m)$-dimensional harmonic Kählerian space H^n, its characteristic function $f(\Omega)$ satisfies the inequality*

$$\dot{f}^2(0) + \frac{5(m+1)^2}{m+7}\ddot{f}(0) \leqq 0 . \tag{1.3}$$

The equality sign is valid if and only if H^n is of constant holomorphic curvature.

In §2, we give some preliminaries. In §3, $\dddot{f}(0)$ is calculated in terms