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SASAKIAN ϕ-SYMMETRIC SPACES*

TOSHIO TAKAHASHI

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1. Introduction. It is known that a Sasakian manifold which is at the same time a locally symmetric space is a space of constant curvature (Okumura [6]). This fact means that a symmetric space condition is too strong for a Sasakian manifold. In this note, we introduce a notion of Sasakian ϕ-symmetric space which is an analogous notion of Hermitian symmetric space, and discuss about its properties.

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2. Definition of Sasakian locally ϕ-symmetric space. Let M be a $(2n+1)$-dimensional Sasakian manifold with structure tensors ϕ, ξ, η and g:

$$(2.1)\qquad \begin{cases} \phi^2 X = -X + \eta(X)\xi \\ \eta(\xi) = 1 \end{cases}$$

$$(2.2)\qquad \begin{cases} g(X, \xi) = \eta(X) \\ g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \end{cases}$$

$$(2.3)\qquad \begin{cases} d\eta(X, Y) = g(\phi X, Y) \\ (\nabla_X \phi) Y = \eta(Y)X - g(X, Y)\xi\ , \end{cases}$$

where ∇ is the Riemannian connection for g and X, Y are tangent vectors on M. Let $\tilde{U}$ be a small open neighborhood of $x \in M$ such that the induced Sasakian structure on $\tilde{U}$, denoted by the same letters, is regular. Let $\pi: \tilde{U} \to U = \tilde{U}/\xi$ be a (local) fibering, and let $(J, \bar{g})$ be the induced Kählerian structure on U (cf. Tanno-Baik [10], Ogiue [5]). Let R and $\bar{R}$ be the curvature tensors constructed by g and $\bar{g}$, respectively. For a vector field $\bar{X}$ on U, we denote its horizontal lift (with respect to the connection from η) by $\bar{X}^*$. Then we have, for any vector fields $\bar{X}$, $\bar{Y}$ and $\bar{Z}$ on U,

$$(2.4)\qquad (\bar{\nabla}_{\bar{X}} \bar{Y})^* = \nabla_{\bar{X}^*} \bar{Y}^* - \eta(\nabla_{\bar{X}^*} \bar{Y}^*)\xi\ ,$$

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