Tôhoku Math. Journ.
29 (1977), 81-89.

THE DEGREE OF CONVERGENCE OF POSITIVE LINEAR OPERATORS

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(Received December 1, 1975)

1. Introduction. Let F be a locally convex Hausdorff space over the field of real numbers and F' its dual space. Let X be a compact convex subset of F, and let $C(X)$ denote the Banach lattice of all real-valued continuous functions on X with the supremum norm $\|\cdot\|$. Let F'_X denote the set of all restrictions of functions in F' to X and 1 the unit function on X. Throughout this paper, (L_α) will be a net of positive linear operators of $C(X)$ into itself.

The following theorem is a generalization of the well-known theorem of Bohman-Korovkin [6]. See, for instance, [1], [2], [5], [7] and [8]:

THEOREM A. *Suppose that $(L_\alpha(g^i))$ converges to g^i for all g in F'_X and for $i = 0, 1, 2$. Then $(L_\alpha(f))$ converges to f for all f in $C(X)$.*

The purpose of this paper is to recast Theorem A in a quantitative form which estimates the rate of convergence of $(L_\alpha(f))$ to f in $C(X)$ in terms of the quantities associated with the system $\{g^i; g \in F'_X, i = 0, 1, 2\}$.

In the case that F is the m-dimensional real Euclidean space R^m our results will cover the results of O. Shisha and B. Mond [10], E. Censor [3] and the result of R. De Vore [4] concerning the estimate for approximation of differentiable functions on the closed interval in the real line. Furthermore, they will suggest the characterization of the saturation class of positive linear operators satisfying the hypotheses of Theorem 1 in the author [7] and that of the Bernstein-Schnabl operators constructed by M. W. Grossman [5].

2. Definitions and lemmas. We shall begin with the following, which all the derived estimates for $\|L_\alpha(f) - f\|$ recasting Theorem A will involve.

DEFINITION 1. Let $\{g_1, \cdots, g_k\}$ be a finite subset of F'_X, δ a non-negative real number and f in $C(X)$. Then we define

$$\omega(f; g_1, \cdots, g_k, \delta) = \sup\{|f(x) - f(y)|; \\ |g_i(x) - g_i(y)| \leqq \delta,\ x, y \in X,\ i = 1, 2, \cdots, k\}$$

and