Tôhoku Math. Journ.
29 (1977), 217-225.

ON THE SEMI-SIMPLE GAMMA RINGS

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(Received January 13, 1976)

1. Introduction. N. Nobusawa [1] introduced the notion of a Γ-ring, more general than a ring, and proved analogues of the Wedderburn-Artin theorems for simple Γ-rings and for semi-simple Γ-rings; Barnes [2] obtained analogues of the classical Noether-Lasker theorems concerning primary representations of ideals for Γ-rings; Luh [3], [4] gave a generalization of the Jacobson structure theorems for primitive Γ-rings having minimum one-sided ideals, and obtained several other structure theorems for simple Γ-rings; Coppage-Luh [5] introduced the notion of Jacobson radical, Levitzki nil radical, nil radical and strongly nilpotent radical for Γ-rings and Barnes' [2] prime radical was studied further. Also, inclusion relations for these radicals were obtained, and it was shown that the radicals all coincide in the case of a Γ-ring which satisfies the descending chain condition on one-sided ideals. The author [6] gave a characterization of the prime radical of a Γ-ring M by introducing the notion of semi-primeness, and obtained close radical properties between a Γ-ring M and its right operator ring R.

In this paper, first we introduce the notion of a Γ-ring M-module and define Jacobson radical $J(M)$ along with the ideas of irreducible modules, while in [5] and [6] $J(M)$ was defined by the ideas of rqr elements. Properties of $J(M)$ and its relation with $J(R)$ are considered here, and it is also shown that our definition coincides with the one in [5] and [6]. After the semi-simplicity is defined by $J(M) = (0)$, the relation between semi-simple M and semi-simple R is considered. Defining the direct sum of Γ-rings S_i, $i \in \mathfrak{A}$, and the primitivity and getting the analogous results of corresponding part in ring theory, we have that a Γ-ring is semi-simple if and only if it is isomorphic to a subdirect sum of primitive Γ-rings.

For all notions relevant to ring theory we refer to [7].

2. Preliminaries. Let M and Γ be additive abelian groups. If for all $a, b, c \in M$ and $\gamma, \delta \in \Gamma$ the following conditions are satisfied, (1) $a\gamma b \in M$, (2) $(a + b)\gamma c = a\gamma c + b\gamma c$, $a(\gamma + \delta)b = a\gamma b + a\delta b$, $a\gamma(b + c) = a\gamma b + a\gamma c$ (3) $(a\gamma b)\delta c = a\gamma(b\delta c)$, then M is called a Γ-ring. If A and B are subsets of a Γ-ring M and $\Theta \subseteq \Gamma$, we denote $A\Theta B$, the subset of M consisting