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# TOEPLITZ OPERATORS ON STRONGLY PSEUDOCONVEX DOMAINS IN STEIN SPACES

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**0. Introduction.** In this paper we study the $C^*$-algebra generated by the Toeplitz operators defined on strongly pseudoconvex domains in normal Stein spaces. We show that there exist short exact sequences of *-algebras which give elements of Ext. defined by Brown-Douglas-Fillmore ([4]).

Let $\Omega$ be a strongly pseudoconvex domain in a normal Stein space $M$ (with or without singularities). Suppose that $\Omega$ has a volume form. Let $L^2(\Omega)$ (resp. $L^2(\partial\Omega)$) be the square integrable functions on $\Omega$ (resp. on $\partial\Omega$) and let $H^2(\Omega)$ (resp. $H^2(\partial\Omega)$) be the holomorphic square integrable functions on $\Omega$ (resp. be the closure of the $C^\infty$-functions on $\partial\Omega$ which are extendible to holomorphic functions in $\Omega$). Let

$$\Pi: L^2(\Omega)) \longrightarrow H^2(\Omega)$$
$$(\text{or } \Pi: L^2(\partial\Omega) \longrightarrow H^2(\partial\Omega))$$

be the orthogonal projection.

For any topological space $X$, we denote by $C(X)$ the Banach algebra of all complex valued continuous functions on $X$, endowed with supremum norm.

For $\phi \in C(\bar{\Omega})$ (resp. $\phi \in C(\partial\Omega)$), we define the Toeplitz operator

$$T_\phi[\Omega]: H^2(\Omega) \longrightarrow H^2(\Omega)$$
$$(\text{resp. } T_\phi[\partial\Omega]: H^2(\partial\Omega) \longrightarrow H^2(\partial\Omega))$$

by $T_\phi(f) = \Pi(\phi\cdot f)$.

Let $\mathscr{T}(\Omega)$ (resp. $\mathscr{T}(\partial\Omega)$) denote the $C^*$-algebra generated by the operators $T_\phi$ for all $\phi$ in $C(\bar{\Omega})$ (resp. $C(\partial\Omega)$). Let us define a mapping

$$\xi: C(\bar{\Omega}) \longrightarrow \mathscr{T}(\Omega)$$
$$(\text{resp. } C(\partial\Omega) \longrightarrow \mathscr{T}(\partial\Omega))$$

by $\xi\phi = T_\phi$, then $\xi$ is contractive and *-linear. For any Hilbert space $H$, we denote by $\mathscr{L}(H)$ the $C^*$-algebra of all bounded linear operators on $H$, by $\mathscr{L}\mathscr{C}(H)$ the closed ideal of compact operators on $H$.

Our main results are as follows.