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Γ-FOLIATIONS AND SEMISIMPLE FLAT HOMOGENEOUS SPACES

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Introduction. In this paper we shall study characteristic classes of Γ-foliations. Our object is to prove a strong vanishing theorem for Pontrjagin classes of the normal bundles of certain Γ-foliations.

Let Γ be a pseudogroup acting on a smooth manifold B of dimension q. A Γ-foliation of codimension q on a smooth manifold M is by definition a maximal family $\mathscr{F}$ of submersions

$$f_\alpha : U_\alpha \to B$$

of open sets U_α in M such that the family $\{U_\alpha\}_\alpha$ is an open covering of M and for each $x \in U_\alpha \cap U_\beta$ there exists an element $\gamma^x_{\beta\alpha} \in \Gamma$ with $f_\beta = \gamma^x_{\beta\alpha} \circ f_\alpha$ in some neighborhood of x. The kernels of the differentials $(f_\alpha)_*$ of submersions f_α then constitute a subbundle $\tau(\mathscr{F})$ of the tangent bundle TM of M. The quotient bundle $\nu(\mathscr{F}) = TM/\tau(\mathscr{F})$ is called the normal bundle of $\mathscr{F}$. Let $\mathrm{Pont}^*(\nu(\mathscr{F}))$ denote the subalgebra of $H^*(M; \boldsymbol{R})$ generated by the real Pontrjagin classes of $\nu(\mathscr{F})$. Then the Bott vanishing theorem [3, 4] states that

$$\mathrm{Pont}^k(\nu(\mathscr{F})) = 0 \quad \text{for} \quad k > 2q ,$$

$\mathrm{Pont}^k(\nu(\mathscr{F}))$ denoting the k-dimensional homogeneous part of $\mathrm{Pont}^*(\nu(\mathscr{F}))$. This gives a sharp bound for general Γ-foliations (Thurston [20]).

On the other hand, Pasternack [13] proved a strong vanishing theorem

$$(*) \qquad \mathrm{Pont}^k(\nu(\mathscr{F})) = 0 \quad \text{for} \quad k > q ,$$

for riemannian foliations $\mathscr{F}$, Γ-foliations with Γ consisting of local isometries of a riemannian structure on B. In the previous paper [11] we improved his result by proving a strong vanishing theorem for conformal or projective foliations.

The purpose of this paper is to extend these results. We thereby obtain the following generalization of the strong vanishing theorem.

MAIN THEOREM. *Let L/L_0 be a semisimple flat homogeneous space of dimension q associated with a semisimple graded Lie algebra $\mathfrak{l} = \mathfrak{g}_{-1} +$*

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