Tôhoku Math. Journ.
31 (1979), 187-194.

DIMENSION OF COHOMOLOGY SPACES OF INFINITESIMALLY DEFORMED KLEINIAN GROUPS

MASAMI NAKADA

(Received May 15, 1978, revised September 2, 1978)

1. Introduction. Let G be the group of all Möbius transformations of $\hat{C} = C \cup \{\infty\}$ of the form $t \mapsto (\alpha t + \beta)/(\gamma t + \delta)$, where $\alpha, \beta, \gamma, \delta \in C$ and $\alpha\delta - \beta\gamma = 1$. Here C is the complex plane. An element $g: t \mapsto (\alpha t + \beta)/(\gamma t + \delta)$, not being the identity, of G is called parabolic if $\mathrm{tr}^2 g = (\alpha + \delta)^2 = 4$.

Let Γ be a subgroup of G and let E be a finite dimensional complex vector space. Let χ be an anti-homomorphism of Γ into $GL(E)$, the group of all non-singular linear mappings of E onto itself. A mapping $z: \Gamma \to E$ is called a cocycle if

$$z(g_1 \circ g_2) = \chi(g_2)(z(g_1)) + z(g_2)$$

for all g_1 and g_2 in Γ. A cocycle z is a coboundary if

$$z(g) = \chi(g)(X) - X$$

for some $X \in E$. We denote by $Z^1_\chi(\Gamma, E)$ the space of all cocycles and by $B^1_\chi(\Gamma, E)$ the space of all coboundaries. A cocycle z is called a parabolic cocycle if, for any parabolic cyclic subgroup Γ_0 of Γ, $z|_{\Gamma_0}$ is an element of $B^1_\chi(\Gamma_0, E)$. We denote by $PZ^1_\chi(\Gamma, E)$ the space of all parabolic cocycles.

The group G is a complex 3-dimensional Lie group isomorphic to $SL(2, C)$ modulo its center. The Lie algebra $\mathfrak{g}$ of G is therefore the algebra of 2×2 complex matrices of trace zero. We identify $\mathfrak{g}$ with the tangent space of G at the identity element e of G.

The adjoint representation Ad of G in $\mathfrak{g}$ is defined by $\mathrm{Ad}(g)(X) = (dA_g)_e(X)$, where $X \in \mathfrak{g}$ and $(dA_g)_e$ is the differential at e of the mapping $A_g: G \ni h \mapsto g^{-1} \circ h \circ g \in G$. The adjoint representation is an anti-homomorphism of G into $GL(\mathfrak{g})$. Hence, for a subgroup Γ of G, we can construct the space of parabolic cocycles $PZ^1_{\mathrm{Ad}}(\Gamma, \mathfrak{g})$.

Let Γ be a subgroup of G and let $\theta: \Gamma \mapsto G$ be a homomorphism of Γ into G. We say that θ is a parabolic homomorphism if $\mathrm{tr}^2\theta(g) = 4$ for any parabolic element g in Γ.

In this paper we prove the following: