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COMPACT COMPLEX SURFACES CONTAINING GLOBAL STRONGLY PSEUDOCONVEX HYPERSURFACES

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Let X be a compact complex manifold of dimension $n \geqq 2$. An open subset N of X is called a *spherical shell*, if N is biholomorphic to

$$S_\varepsilon = \{z \in \boldsymbol{C}^n : 1 - \varepsilon < \|z\| < 1 + \varepsilon\}$$

for some ε, $0 < \varepsilon < 1$, where $\|z\|$ denotes the standard complex Euclidean norm of a vector $z = (z_j)$ in the n-dimensional complex vector space $\boldsymbol{C}^n$, i.e., $\|z\|^2 = \sum_{j=1}^n |z_j|^2$. N is called a *global spherical shell* (abbrev., GSS), if $X - N$ is connected. In [4], we have proved that *a compact complex manifold containing a* GSS *is biholomorphic to a deformation of a modification of a primary Hopf manifold at finitely many points*. In this paper we restrict ourselves to the case of surfaces, i.e., $n = 2$, and consider compact complex surfaces containing (real analytic) *global strongly pseudoconvex hypersurfaces* (GSPH) *which bound Stein domains possibly with finitely many isolated singular points*. Then we can determine all such surfaces (Theorem).

Here we shall use the definitions and some results in Rossi [5, 6]. Let Σ be a compact real analytic CR-hypersurface with $\dim_{\boldsymbol{R}} \Sigma = 3$. It is known that Σ admits a realization as a real hypersurface in a complex *manifold* of (complex) dimension 2. Namely, there exist a complex manifold M of dimension 2 and a CR-injection $j : \Sigma \to M$ such that the CR-structure on Σ coincides with the CR-structure induced from M. Moreover, if $j_i : \Sigma \to M_i$ $(i = 1, 2)$ are two realizations of Σ, then $j_2 \circ j_1^{-1}$ extends to a biholomorphic mapping between small neighborhoods of $j_i(\Sigma)$ ([6]). This implies that *the realization of Σ is unique as a germ*.

We say that Σ *bounds a Stein domain*, if there exist a (reduced irreducible) complex space $\hat{M}$, a subdomain M of $\hat{M}$ which is free from singular points, and a realization $j : \Sigma \to M$ of Σ, such that $j(\Sigma)$ bounds a relatively compact Stein open subset D of $\hat{M}$. Note that D may have finitely many isolated singular points. We remark that there exist strongly pseudoconvex hypersurfaces Σ with $\dim_{\boldsymbol{R}} \Sigma = 3$ such that Σ do *not* bound any Stein domains ([5]).

Let S be a compact complex manifold of dimension 2, which will be