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HOLOMORPHIC FAMILIES OF RIEMANN SURFACES AND TEICHMÜLLER SPACES II

Applications to the uniformization of algebraic surfaces and the compactification of two dimensional Stein manifolds

YOICHI IMAYOSHI

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Introduction. Let $\bar{\mathscr{S}}$ be a two dimensional complex manifold and let C be a non-singular one dimensional analytic subset of $\bar{\mathscr{S}}$ or an empty set. Denote by D the unit disc $|t| < 1$ and by D^* the punctured unit disc $0 < |t| < 1$ in the complex t-plane. We assume that a proper holomorphic mapping $\bar{\pi}: \bar{\mathscr{S}} \to D^*$ satisfies the following two conditions;

1) $\bar{\pi}$ is of maximal rank at every point of $\bar{\mathscr{S}}$, and

2) by setting $\mathscr{S} = \bar{\mathscr{S}} - C$ and $\pi = \bar{\pi}|\mathscr{S}$, the fibre $S_t = \pi^{-1}(t)$ of $\mathscr{S}$ over each $t \in D^*$ is an irreducible analytic subset of $\mathscr{S}$ and is of fixed finite type (g, n) with $2g - 2 + n > 0$ as a Riemann surface, where g is the genus of S_t and n is the number of punctures of S_t. We call such a triple $(\mathscr{S}, \pi, D^*)$ a holomorphic family of Riemann surfaces of type (g, n) over D^*. We also say that $\mathscr{S}$ has a holomorphic fibration $(\mathscr{S}, \pi, D^*)$ of type (g, n).

Our main problem is to construct a completion of $(\mathscr{S}, \pi, D^*)$ canonically in such a way that the central fibre is a Riemann surface (possibly with nodes) of the same type (g, n) modulo a finite group of automorphisms.

As a continuation of the preceeding paper [6], we treat the completion of $(\mathscr{S}, \pi, D^*)$ in the first half of this paper. For a holomorphic family $(\mathscr{S}, \pi, D^*)$ of Riemann surfaces of type (g, n) with $2g - 2 + n > 0$, we regard the fibre S_t over $t \in D^*$ as a point $\Phi(t)$ in a Teichmüller space. It should be noted that, in general, Φ is a multi-valued analytic mapping. In §1 and §2, we recall terminologies and notations in [6]. In §3, we study the behavior of Φ as t tends to zero. In §4, using the result of §3, we canonically construct a completion $(\hat{\mathscr{S}}, \hat{\pi}, D)$ of $(\mathscr{S}, \pi, D^*)$ and, in §5, we prove an extension theorem for a holomorphic mapping F of $\mathscr{S}$ into $\hat{\mathscr{S}}$ with $\pi = \hat{\pi} \circ F$.

In the second half of this paper, as applications of the above results,