Tôhoku Math. Journ.
32 (1980), 9–34.

SRH-DECOMPOSITIONS OF CODIMENSION-ONE FOLIATIONS AND THE GODBILLON-VEY CLASSES

TOSHIYUKI NISHIMORI

(Received December 22, 1978, revised April 13, 1979)

1. Introduction. In this paper we show that the Godbillon-Vey classes of codimension-one foliations with a certain qualitative property are zero.

Since the Godbillon-Vey class was defined in Godbillon-Vey [1], many authors have published studies on it. Thurston [14] proved that the Godbillon-Vey class gives rise to a surjective homomorphism

$$gv\colon \mathscr{F}\Omega^{\infty}_{3,1} \longrightarrow \boldsymbol{R}$$

where $\mathscr{F}\Omega^{\infty}_{3,1}$ is the foliated cobordism group of transversely oriented codimension-one foliations of closed oriented 3-manifolds. The problem to determine its kernel is still open. (See Problem 4 in Lawson [4]). In this point of view it is interesting to investigate what type of foliations are contained in the kernel of gv. Herman [3] proved that a foliation of the 3-torus whose leaves are diffeomorphic to $\boldsymbol{R}^2$ is in the kernel of gv.

On the other hand, the author has been studying the qualitative theory in [8]–[11] and saw that codimension-one foliations with a certain qualitative property admit nice decompositions. By making use of these decompositions, we can compute the Godbillon-Vey classes.

The main result is the following.

THEOREM 1. *Let $\mathscr{F}$ be a transversely orientable codimension-one C^∞ foliation of a closed orientable manifold M. Suppose that the depth $d(\mathscr{F})$ of $\mathscr{F}$ is finite and all holonomy groups of $\mathscr{F}$ are abelian. Then we have*

(1) *If* $\dim M = 3$, *then* $gv(\mathscr{F}) = 0$.

(2) *Let* $\dim M > 3$. *If, for each leaf F of $\mathscr{F}$ whose holonomy group is non-trivial, the cohomology group $H^2_{\mathrm{comp}}(F\colon \boldsymbol{R})$ with compact support is trivial, then* $gv(\mathscr{F}) = 0$.

The author conjectures that the condition in (2) of Theorem 1 is not essential.

The author was partially supported by the Sakkokai Foundation.