Tôhoku Math. Journ.
32 (1980), 375-392.

FOLIATED COBORDISMS OF SUSPENDED FOLIATIONS

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(Received May 10, 1979, revised September 21, 1979)

Introduction. Let $(M^n, \mathscr{F})$ be a codimension q foliation on a closed oriented n-manifold M^n. Then the suspended foliation $\Sigma_f(M^n, \mathscr{F})$ is obtained from $(M^n, \mathscr{F}) \times [0, 1]$ by the identification of $(M^n, \mathscr{F}) \times \{0\}$ with $(M^n, \mathscr{F}) \times \{1\}$ by means of a foliation preserving diffeomorphism f. There are interesting examples of foliations of this type. For example, let $\mathscr{F}$ be a codimension 1 foliation on T^{n+1} whose leaves are transverse to the fibers of the canonical fibration $S^1 \to T^{n+1} = S^1 \times T^n \to T^n$. Then we can construct $\mathscr{F}$ from mutually commuting automorphisms $f_1, \cdots, f_n$ of S^1 (cf. Herman [5]). We denote $(T^{n+1}, \mathscr{F})$ by $\mathscr{F}(f_1, \cdots, f_n)$. Then it is easy to see that $\mathscr{F}(f_1, \cdots, f_n)$ is a suspended foliation of $\mathscr{F}(f_1, \cdots, \hat{f}_i, \cdots, f_n)$ by f_i, where the "hat" means that the term is left out.

Recently, Herman [5] and Morita-Tsuboi [22] proved that the Godbillon-Vey class of $\mathscr{F}(f_1, \cdots, f_n)$ is zero. Considering the conjecture that the map $\mathrm{GV}: \mathscr{F}\Omega_1(3) \to \boldsymbol{R}$ given by the Godbillon-Vey number is injective, we may ask if $\mathscr{F}(f_1, \cdots, f_n)$ is foliated null-cobordant. However, this seems to be very difficult even for the case $\mathscr{F}(f, g)$ on T^3 (see Tsuboi [23]). Moreover, $\Sigma_f(M, \mathscr{F})$ may not be null-cobordant in general even if $(M, \mathscr{F})$ is null-cobordant and $f \in \mathrm{LD}(M, \mathscr{F})$, where $\mathrm{LD}(M, \mathscr{F})$ is the group of all leaf preserving diffeomorphisms of $(M, \mathscr{F})$. We will give such an example in §6. But it seems to be natural to conjecture that $\Sigma_f(M, \mathscr{F})$ is null-cobordant for $f \in \mathrm{FD}(M, \mathscr{F})_0$, where $\mathrm{FD}(M, \mathscr{F})_0$ is the identity component of the group of all foliation preserving diffeomorphisms of $(M, \mathscr{F})$, because the elements in $\mathrm{FD}(M, \mathscr{F})_0$ are considerably restricted (see Lemma 10 in §4).

In this paper we consider this problem and verify the above conjecture for some codimension 1 foliations, i.e., in §3 for the Reeb foliation $(S^3, \mathscr{F}_R)$ and a modified Reeb foliation $(S^3, \bar{\mathscr{F}}_R)$, in §4 for the foliation $(S^3, \mathscr{F}_a)$ with the Godbillon-Vey number of $(S^3, \mathscr{F}_a) = a \neq 0$ constructed by Thurston [20], and in §2 for the foliation defined by a non-vanishing smooth closed 1-form. Concerning the last foliation, we will show that $\Sigma_f(M, \mathscr{F})$ is null-cobordant for $f \in \mathrm{FD}(M, \mathscr{F}) \cap \mathrm{Diff}^\infty_+(M)_0$. These results give some information on the relation between $\Sigma_f(M^3, \mathscr{F})$