Tôhoku Math. Journ.
32 (1980), 607-613.

PRACTICAL STABILITY AND LYAPUNOV FUNCTIONS*

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

STEPHEN R. BERNFELD AND V. LAKSHMIKANTHAM

(Received November 5, 1979, revised January 5, 1980)

1. Introduction. The notion of "practical stability" was discussed in the monograph by LaSalle and Lefschetz [6] in which they point out that stability investigations may not assure "practical stability" and vice versa. For example an aircraft may oscillate around a mathematically unstable path, yet its performance may be acceptable. Motivated by this, Weiss and Infante introduced the concept of finite time stability [7]. They were interested in the behavior of systems contained within specified bounds during a fixed time interval. Many problems fall into this category including the travel of a space vehicle between two points and the problem, in a chemical process, of keeping the temperature within certain bounds.

In particular, Weiss and Infante [7] provided sufficient conditions for finite time stability in terms of Lyapunov functions. Moreover, Weiss [9] provided necessary and sufficient conditions for uniform finite time stability and exponential contractive stability. These results were extended by Kayande [3] who obtained necessary and sufficient conditions for contractive stability (without requiring the exponential behavior assumed in [9]).

The sufficiency part of the above results were extended by Kayande and Wong [4], and Gunderson [1], who applied the comparison principle. Moreover Hallam and Komkov [2] generalized the concept of the finite time stability of the zero solution to that of arbitrary closed sets.

In this paper we analyze a more general notion of practical stability than is provided for by finite time stability considerations. Our state space includes finite as well as infinite dimensional Banach spaces. The sets upon which we impose our stability conditions are not restricted to balls containing the origin as is done by the others. This leads to interesting implications. We first present necessary and sufficient conditions for generalized practical stability, in a more meaningful setting than that of Kayande [3] for finite time stability. We then apply our

* Research partially supported by U. S. Army Grant DAAG-29-77-G0062.