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ANALYTIC AUTOMORPHISMS OF THE COMPLEMENT OF AN ALGEBRAIC CURVE IN THE COMPLEX PROJECTIVE PLANE

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Introduction. Every analytic automorphism of a compact Riemann surface punctured at a finite number of points is analytically continued to an automorphism of the compact Riemann surface. On the contrary, there are many examples of compact complex analytic surface S and analytic curve A in S such that the complement $S\setminus A$ has an analytic automorphism which cannot be continued to a bimeromorphic transformation of S. Such an analytic automorphism will be called a *transcendental automorphism* of $S\setminus A$ in this paper. By Sakai [5], the logarithmic Kodaira dimension of $S\setminus A$ having a transcendental automorphism is smaller than two. On the other hand, Wakabayashi [8] has given some necessary conditions on algebraic curves A in the complex projective plane $\boldsymbol{P}^2$ under which the logarithmic Kodaira dimension of $\boldsymbol{P}^2\setminus A$ is smaller than two. In this paper, we show that $\boldsymbol{P}^2\setminus A$ having a transcendental automorphism is very special, in the following sense:

A rational function f on a non-singular complex algebraic surface S is called a rational function *of special type* on S if the irreducible components of almost all level curves $f=$ const. of f in $S\setminus\{$the indetermination points of $f\}$ are biholomorphically equivalent to the Gaussian plane $\boldsymbol{C}$ or to the punctured Gaussian plane $\boldsymbol{C}^*=\boldsymbol{C}\setminus\{0\}$. We can present our principal result as follows (see also Theorem 2 in §4): If $\boldsymbol{P}^2\setminus A$ has a transcendental analytic automorphism and if A is not a non-singular cubic curve, then there exists a rational function f on $\boldsymbol{P}^2$ such that the restriction $f|_{\boldsymbol{P}^2\setminus A}$ to $\boldsymbol{P}^2\setminus A$ is a rational function of special type on $\boldsymbol{P}^2\setminus A$. If, furthermore, A is irreducible, then A is a level curve of the rational function of special type on $\boldsymbol{P}^2$.

Our principle is as follows: If $\boldsymbol{P}^2\setminus A$ has a transcendental automorphism, then there exists a holomorphic mapping φ of the punctured disc into $\boldsymbol{P}^2\setminus A$ with an essential singularity at the origin whose cluster set $\varphi(0;\boldsymbol{P}^2)$ at the origin in $\boldsymbol{P}^2$ is contained in A. After the minimal resolution of the singularities of A and its normally-crossing minimalization (σ_1 in §2, 2°), we apply a theorem on the cluster sets due to Nishino and Suzuki [4] to our problem (cf. §2, 1°).