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REAL ANALYTIC $SL(n, R)$ ACTIONS ON SPHERES

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0. Introduction. Let $SL(n, \boldsymbol{R})$ denote the group of all $n \times n$ real matrices of determinant 1. In the previous paper [12], we classified real analytic $SL(n, \boldsymbol{R})$ actions on the standard n-sphere for each $n \geqq 3$. In this paper we study real analytic $SL(n, \boldsymbol{R})$ actions on the standard m-sphere for $5 \leqq n \leqq m \leqq 2n-2$. We shall show that such an action is characterized by a certain real analytic $\boldsymbol{R}^\times$ action on a homotopy $(m-n+1)$-sphere. Here $\boldsymbol{R}^\times$ is the multiplicative group of all non-zero real numbers.

In Section 1 we construct a real analytic $SL(n, \boldsymbol{R})$ action on the standard $(n+k-1)$-sphere from a real analytic $\boldsymbol{R}^\times$ action on a homotopy k-sphere satisfying a certain condition for each $n+k \geqq 6$. In Section 3 we state a structure theorem for a real analytic $SL(n, \boldsymbol{R})$ action which satisfies a certain condition on the restricted $SO(n)$ action, and in Section 5 we state a decomposition theorem and a classification theorem. In Section 6 we construct real analytic $\boldsymbol{R}^\times$ actions on the standard k-sphere. It can be seen that there are infinitely many (at least the cardinality of the real numbers) mutually distinct real analytic $SL(n, \boldsymbol{R})$ actions on the standard m-sphere.

1. Construction. Let $\psi: \boldsymbol{R}^\times \times \Sigma \to \Sigma$ be a real analytic $\boldsymbol{R}^\times$ action on a real analytic closed manifold Σ which is homotopy equivalent to the k-sphere. Define a real analytic involution T of Σ by $T(x) = \psi(-1, x)$ for $x \in \Sigma$. Put $F = F(\boldsymbol{R}^\times, \Sigma)$, the fixed point set. We say that the action ψ satisfies the condition (P) if

(i) there exists a compact contractible k-dimensional submanifold X of Σ such that $X \cup TX = \Sigma$ and $X \cap TX = F$,

(ii) there exists a real analytic $\boldsymbol{R}^\times$ equivariant isomorphism j of $\boldsymbol{R} \times F$ onto an open set of Σ such that $j(0, x) = x$ for $x \in F$. Here $\boldsymbol{R}^\times$ acts on $\boldsymbol{R}$ by the scalar multiplication.

Notice that $F = F(T, \Sigma)$, the fixed point set of the involution T by the condition (i), and hence F is a real analytic $(k-1)$-dimensional closed submanifold of Σ. Define a map

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