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THE HAUSDORFF DIMENSION OF LIMIT SETS OF SOME FUCHSIAN GROUPS

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1. Preliminaries. Let Γ and Λ be a non-elementary finitely generated Fuchsian group of the second kind and its limit set, respectively. Put $M_t(\delta, \Lambda) = \inf \sum_i |I_i|^t$, where the infimum is taken over all coverings of Λ by sequences $\{I_i\}$ of sets I_i with the spherical diameter $|I_i|$ less than a given number $\delta > 0$. Further, put $M_t(\Lambda) = \sup M_t(\delta, \Lambda)$, which is called the t-dimensional Hausdorff measure of Λ. It is shown in [2] that if $\infty \notin \Lambda$, $M_t(\Lambda) = \sup_\delta \inf \sum_i \mathrm{dia}^t(I_i)$, where the infimum is taken over all coverings of Λ by sequences $\{I_i\}$ of sets I_i with the Euclidean diameter $\mathrm{dia}(I_i)$. We call $d(\Lambda) = \inf\{t > 0;\ M_t(\Lambda) = 0\}$ the Hausdorff dimension of Λ. In [3] Beardon proved that $d(\Lambda) < 1$ for the limit set $\Lambda(\not\ni \infty)$ of any finitely generated Fuchsian group of the second kind.

The purpose of this note is to show the continuity of $d(\Lambda)$ with respect to quasiconformal deformations of Γ.

Let w be a K-quasiconformal mapping of the unit disc D onto itself and $w(0) = 0$. The following distortion theorem is due to Mori [5].

PROPOSITION 1. *Let w be a K-quasiconformal mapping of D onto itself and $w(0) = 0$. Then for every pair of points z_1, z_2 with $|z_1| \leqq 1$, $|z_2| \leqq 1$,*

$$|w(z_1) - w(z_2)| < 16|z_1 - z_2|^{1/K}, \quad (z_1 \neq z_2).$$

Let Γ be a finitely generated Fuchsian group acting on D. We say that Γ has a type $(g; n; m)$ if $S = D/\Gamma$ is obtained from a compact surface of genus g by removing j ($\geqq 0$) points, m ($\geqq 0$) conformal discs and if there are finitely many, say k ($\geqq 0$), ramification points on S, where $n = j + k$. Suppose that to each ramification point a_i ($i = 1, 2, \cdots, k$) on S, there is assigned an integer ν_i, $1 < \nu_1 \leqq \nu_2 \leqq \cdots \leqq \nu_k < +\infty$. Then we say that Γ has the signature $(g; \nu_1, \nu_2, \cdots, \nu_k, \nu_{k+1}, \cdots, \nu_n; m)$, where $\nu_{k+1} = \cdots = \nu_{n-1} = \nu_n = \infty$. We call an isomorphism χ of a Fuchsian group Γ_0 onto Γ_1 quasiconformal if there exists a quasiconformal mapping w which maps D onto itself and $w(0) = 0$ such that $\chi(A) = wAw^{-1}$ for all $A \in \Gamma_0$. The following proposition was proved by Bers [4].