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INEQUALITIES OF FEJÉR-RIESZ TYPE FOR HOLOMORPHIC FUNCTIONS ON CERTAIN PRODUCT DOMAINS

Dedicated to the memory of Professor Teishirô Saitô

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1. Introduction. Let f be a holomorphic function in a neighborhood of the closed unit disc in the complex plane C and l be a chord of the boundary circle C. Then the following inequality holds for every p, $0 < p < \infty$:

$$\int_l |f(z)|^p |dz| \leqq K_l \int_C |f(z)|^p |dz| \tag{1}$$

in which K_l is a constant depending only on l, and $K_l < 1$ ([1], [5]). If l coincides with a diameter of the disc, then $K_l = 1/2$ and the Fejér-Riesz inequality follows ([2]), and this is extended to the H^p-functions on the unit ball of C^n, $n \geqq 2$ ([4]).

The purpose of the present note is to obtain an inequality similar to (1) for the H^p-functions on a domain in C^N which is a product of balls in C^{n_j}, $j = 1, \cdots, m$. This inequality gives, as a special case, an extension of (1) to H^p-functions on the unit polydisc in C^n which is not treated in [4], and we note that the constant appearing in the inequality exhibits a remarkable contrast to that for the unit ball.

2. Statements of results. Let $C^N = C^{n_1} \times \cdots \times C^{n_m}$ and let $Z = (Z^1, \cdots, Z^m) \in C^N$, where we shall use the notations $Z^j = (z_1^j, \cdots, z_{n_j}^j) \in C^{n_j}$ and $X^j = (x_1^j, x_2^j, \cdots, x_{2n_j-1}^j, x_{2n_j}^j) \in R^{2n_j}$ with $z_k^j = x_{2k-1}^j + i x_{2k}^j$, $k = 1, \cdots, n_j$; $j = 1, \cdots, m$. We shall write $\|Z^j\|^2 = |z_1^j|^2 + \cdots + |z_{n_j}^j|^2$ and $\|X^j\|^2 = (x_1^j)^2 + \cdots + (x_{2n_j}^j)^2$. If $A^j = (a_1^j, \cdots, a_{2n_j}^j) \in R^{2n_j}$, we write $A^j X^j = a_1^j x_1^j + \cdots + a_{2n_j}^j x_{2n_j}^j$. We consider a domain $B = B_1 \times \cdots \times B_m$ in C^N, where B_j is the unit ball in C^{n_j} centered at the origin, i.e., B_j is the set of points Z^j such that $\|Z^j\| < 1$. We let ∂B stand for the Bergman-Šilov boundary of B, $\partial B = \partial B_1 \times \cdots \times \partial B_m$, where ∂B_j is the boundary of B_j. We denote the Lebesgue measure on ∂B by $d\tau$; more precisely, this means that $d\tau$ is the product measure of elements of the surface area of spheres ∂B_j, $j = 1, \cdots, m$. The Hardy space $H^p(B)$, $0 < p < \infty$, is defined and properties we need can be derived as in the case of polydiscs ([6]); especially,