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# EXISTENCE OF SOLUTIONS AND GALERKIN APPROXIMATIONS FOR NONLINEAR FUNCTIONAL EVOLUTION EQUATIONS

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1. **Introduction—Preliminaries.** In this paper we are concerned with existence and approximation results for nonlinear functional evolution equations in Banach spaces. Let $X$ be a Banach space with norm $\|\cdot\|$, and let $C = C([-r, 0], X)$ be the Banach space of continuous functions mapping the interval $[-r, 0]$, for some $r > 0$, into $X$ with norm $\|\psi\|_C = \sup_{\theta\in[-r,0]}\|\psi(\theta)\|$. Let $x_t \in C$ be defined by $x_t(\theta) = x(t+\theta)$ for $\theta\in[-r, 0]$. In [9] we examined the existence of a unique strong solution of the abstract initial value problem

$$\text{(FDE)}\qquad x'(t) + A(t)x(t) = G(t, x_t)\ , \qquad t\in[0, T]\ , \qquad x_0 = \phi\ ,$$

where $A(t): D(A(t)) = D \subset X \to X$, $G$ satisfies a global Lipschitz condition with respect to both variables, and $\phi \in C$ is such that $\phi' \in C$ and $\phi(0)\in D$. Furthermore, we required that $X^*$, the dual of $X$, be uniformly convex and for each $t\in[0, T]$, $A(t)$ be $m$-accretive (see definition below) and satisfy a Kato time-dependence condition of the form

$$(*)\qquad \|A(t)x - A(s)x\| \leqq |t-s|L(\|x\|)(1 + \|A(s)x\|)$$

for all $t, s\in[0, T]$ and $x \in D$, where $L: R_+ = [0, \infty) \to R_+$ is a given increasing function.

By a "strong solution" of (FDE) on $[0, T]$ we mean an absolutely continuous $X$-valued function which, for almost all $t\in[0, T]$, is strongly differentiable and satisfies (FDE). The unique strong solution $x(t)$ of (FDE), whose existence was known from previous results, was shown in [9] to be the uniform limit of strongly continuously differentiable solutions of approximating equations for (FDE) involving the Yosida approximants of $A(t)$. In [10] a method of lines for the approximation of the solution $x(t)$ of (FDE) was developed.

Our purpose in this paper is two-fold. We first establish a local existence result for a more general nonlinear abstract functional problem of the type:

$$\text{(DE)}\qquad x'(t) + A(t, x_t)x(t) = 0\ , \qquad t\in[0, T)\ , \qquad x_0 = \phi\ ,$$