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UNIFORM INTEGRABILITY OF CONTINUOUS EXPONENTIAL MARTINGALES

Dedicated to Professor Tamotsu Tsuchikura on his sixtieth birthday

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1. Introduction. Given a local martingale M, the associated exponential local martingale Z is not necessarily uniformly integrable. As is first indicated by Girsanov in [2], to know whether Z is a uniformly integrable martingale is very important in certain questions concerning the absolute continuity of probability laws of stochastic processes. However, it seems to us that the essential feature of the problem appears in the case where M is continuous. For that reason, assuming the sample continuity of M we consider the uniform integrability of Z in this paper: but we have no mind to deny the significance of the extension to right continuous martingales. Historically, in the last ten years many sufficient conditions about this problem have successively been found: for example, see Novikov [9, 10], Kazamaki [4, 5], Lépingle and Mémin [7] and Okada [11].

This paper consists of six sections, and Section 5 contains the main result. Our aim here is to give a new sufficient condition which is an improvement of the above-mentioned criteria. In Sections 2 and 3 we collect some notations and technical results that are used in later sections. In Section 4 we shall deal with a special case in order to explain our idea explicitly. Finally, in Section 6 we shall state some remarks on a BMO-martingale in connection with the problem about the uniform integrability of exponential martingales.

2. Preliminaries. Let (Ω, F, P) be a complete probability space with a non-decreasing right continuous family $(F_t)_{0\leqq t<\infty}$ of sub σ-fields of F such that $F = \bigvee_{t\geqq 0} F_t$ and F_0 contains all null sets. It goes without saying that the martingales here are adapted to this filtration.

Given a continuous local martingale M with $M_0 = 0$, consider the exponential local martingale Z defined by the formula

$$Z_t = \exp\left(M_t - \frac{1}{2}\langle M\rangle_t\right) \quad (0 \leqq t < \infty) \tag{1}$$

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