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AN ALGEBRAIC APPROACH TO ISOPARAMETRIC HYPERSURFACES IN SPHERES II

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Introduction. In [1] it was shown that isoparametric hypersurfaces in spheres with 4 distinct principal curvatures can be equivalently described by isoparametric triple systems. These triple systems have a "Peirce decomposition"

$$V = V_{11} \oplus V_{10} \oplus V_{12}^{+} \oplus V_{12}^{-} \oplus V_{22} \oplus V_{20}$$

and every element in a Peirce space V_{ij} is a scalar multiple of a tripotent. Moreover, it was proved that this property essentially characterizes isoparametric triple systems.

In this paper we investigate the Peirce decomposition relative to a tripotent from a Peirce space V_{ij}. We also begin the study of the fine structure of V_{12} by introducing the subspaces Q and JV_{12}.

The results of this paper are used in [2] and [3] and lay the foundations for subsequent publications.

The paper is organized as follows. In §1 we compute all the triple products $\{uvw\}$ where each element u, v, w lies in some Peirce space V_{ij}. In §§2, 3 we compute the Peirce decompositions of V relative to tripotents from V_{10}, V_{20} and V_{12}. Finally, in §4 we introduce the space $Q \subset V_{12}$ and show how it is connected to elements of the dual triple satisfying Jordan composition rules. This space is also important for the investigation of isoparametric triple systems of FKM-type, [2], §8.

For definitions and notations we refer to [1].

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1. Various triple products. In this section we consider an isoparametric triple V. We fix orthogonal tripotents (e_1, e_2) and denote by V_{ij} the Peirce spaces relative to (e_1, e_2). By [1, Remark 4.3. a] the elements $e = \lambda(e_1 + e_2)$ and $\hat{e} = \lambda(e_1 - e_2)$, $\lambda = (\sqrt{2})^{-1}$ are maximal tripotents of V.

1.1. In this subsection we compute the triple products where each

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