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BOUNDEDNESS OF THE BERGMAN PROJECTOR AND BELL'S DUALITY THEOREM

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Introduction. Suppose given a bounded domain Ω in $\boldsymbol{C}^n$ with smooth boundary, and a positive integer s. In case Ω is strictly pseudo-convex, Bell [3] has discussed the duality $W^sH(\Omega) \subset L^2H(\Omega) \subset W^sH(\Omega)^*$, where $W^sH(\Omega)$ denotes the space of holomorphic functions in Ω contained in the $L^2(\Omega)$ Sobolev space $W^s(\Omega)$ of order s. He has shown that $W^sH(\Omega)^* = W^{-s}H(\Omega)$ as a Banach space and that the natural isometry $\Lambda^s\colon W^sH(\Omega) \to W^sH(\Omega)^*$ is given by

$$\Lambda^s g(z) = (g, K(\cdot, z))_s \quad \text{for} \quad g \in W^sH(\Omega) \quad \text{and} \quad z \in \Omega ,$$

where $K(\cdot, \cdot)$ stands for the Bergman kernel. The purpose of the present paper is to observe that such a duality is most naturally stated in connection with the following regularity condition on the Bergman projector K:

$$(\mathrm{R})_0^s \qquad K\colon W_0^s(\Omega) \to W^sH(\Omega) \subset W^s(\Omega) \quad \text{is bounded} .$$

In particular, we shall show that $(\mathrm{R})_0^s$ is equivalent to that $W^sH(\Omega)^* = W_{\mathrm{cl}}^{-s}H(\Omega)$ as a Banach space, where $W_{\mathrm{cl}}^{-s}H(\Omega)$ denotes the closure of $L^2H(\Omega)$ in $W^{-s}(\Omega)$. Also an expression for Λ^s as above will be verified under the assumption $(\mathrm{R})_0^s$.

The regularity condition $(\mathrm{R})_0^s$ is the case without loss of derivatives of the so-called condition R due to Bell [2] (see also Bell-Ligocka [8]), while the condition R has been successfully used in the problem of extending a given biholomorphic mapping smoothly to the boundary, a problem which is expected to be solved affirmatively for pseudo-convex domains (see Fefferman [11]). In particular, Ligocka [23] has given a positive answer to the smooth extension problem for domains satisfying the condition R, see also Bell-Ligocka [8], Bell [4], Bell-Catlin [7], Diederich-Fornaess [10], and the references therein.

Observe that $(\mathrm{R})_0^s$ is satisfied if

$$(\mathrm{R})^s \qquad K\colon W^s(\Omega) \to W^sH(\Omega) \subset W^s(\Omega) \quad \text{is bounded} ,$$

which is another case of the condition R without loss of derivatives and

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