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CURVATURE OPERATOR OF THE BERGMAN METRIC ON A HOMOGENEOUS BOUNDED DOMAIN

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0. Introduction. It is well known that a symmetric bounded domain in a complex Euclidean space possesses the following two curvature properties of the Bergman metric:

(i) The sectional curvature is non-positive.

(ii) When the domain is irreducible, the curvature operator has at most two distinct eigenvalues.

The latter is shown in Calabi and Vesentini [6], and Borel [3]. Recently, it was shown in D'Atri and Miatello [10] that symmetric bounded domains are characterized by the property (i) in the category of homogeneous bounded domains. The main purpose of this paper is to show that symmetric bounded domains are characterized by the property (ii) in the category of irreducible, homogeneous bounded domains (Theorem 7.2). A theorem of this type was obtained by Itoh [15]: A compact, Kähler, simply connected homogeneous space with the second Betti number $b_2 = 1$ is Hermitian symmetric if and only if the curvature operator has at most two distinct eigenvalues. Several characterizations of symmetric bounded domains in the category of homogeneous bounded domains are discussed also in [11], [12].

Our proof of Theorem 7.2 is based on the theory of normal j-algebras. After studying curvature properties of a normal j-algebra in §§3-6, we shall prove Theorem 7.2 in §7. The proof is divided into two steps as follows: Let $(\mathfrak{g}, j)$ be a normal j-algebra corresponding to an irreducible, homogeneous bounded domain D with at most two distinct eigenvalues of the curvature operator, and let $\mathfrak{g} = \sum_{a\leq b} \mathfrak{n}_{ab} + \sum_{a\leq b} j\mathfrak{n}_{ab} + \sum_{a} \mathfrak{n}_{a*}$ be its root space decomposition. We first show that $\dim \mathfrak{n}_{ab} = n_{12}$ for every pair (a, b) with $a < b$, and that $\dim \mathfrak{n}_{a*} = n_{1*}$ for every a (Lemma 7.5). This means that D is quasi-symmetric in the sense of Satake [23] (cf. [10]). We next conclude that D is symmetric, by means of a criterion of Dorfmeister [12] for a quasi-symmetric bounded domain to be symmetric (Proposition 7.8).

Several by-products of our argument are given in §§8-9. Denote by HSC the holomorphic sectional curvature of the Berman metric g on a