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MINIMAL SURFACES IN A SPHERE WITH GAUSSIAN CURVATURE NOT LESS THAN 1/6

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1. Introduction. Let $S^n(r)$ be an n-dimensional sphere in the $(n+1)$-dimensional Euclidean space $\boldsymbol{R}^{n+1}$ with radius r and $x: M \to S^n(r)$ be an isometric minimal immersion of a differentiable 2-manifold M into $S^n(r)$ $(n \geqq 3)$. If M is compact and the Gaussian curvature K of M is non-negative, not identically zero, then the genus of M is zero, by the Gauss-Bonnet theorem. In [3], Borůvka has constructed a series of isometric minimal immersions $\psi_k: S^2((k(k+1)/2)^{1/2}) \to S^{2k}(1)$ by making use of spherical harmonic polynomials of degree k. ψ_2 is the Veronese surface with $K = 1/3$ in $S^4(1)$ and ψ_3 is called the generalized Veronese surface with $K = 1/6$ in $S^6(1)$. Later, in [4], Calabi has proved that, any isometric full minimal immersion of $S^2(K^{-1/2})$ into $S^n(1)$ is congruent to some ψ_k and so there exists an integer k such that $K = 2/k(k+1)$ and $n = 2k$.

On the other hand, Lawson [13] and Benko et al. [2] have proved the following:

THEOREM A. *Let $x: M \to S^n(1)$ be an isometric minimal immersion of a complete, connected, oriented 2-manifold M into $S^n(1) (n \geqq 3)$. If $1/3 \leqq K \leqq 1$, then either $x(M)$ is totally geodesic and $K \equiv 1$, or the Veronese surface in $S^4(1)$ and $K \equiv 1/3$.*

In this paper, we shall prove:

THEOREM B. *Let $x: M \to S^n(1)$ be an isometric minimal immersion of a complete, connected, oriented 2-manifold M into $S^n(1)$ $(n \geqq 3)$. If $1/6 \leqq K \leqq 1$, then either* (1) *$x(M)$ is totally geodesic and $K \equiv 1$,* (2) *the generalized Veronese surface in $S^6(1)$ and $K \equiv 1/6$, or* (3) *a minimal surface in $S^4(1)$ with $1/6 \leqq K \leqq 1$.*

As a corollary to Theorem B, we can prove the following:

COROLLARY C. *If $1/6 \leqq K \leqq 1/3$, then $K \equiv 1/3$ or $1/6$, and either $x(M)$ is the Veronese surface in $S^4(1)$ in the case of $K \equiv 1/3$, or the generalized Veronese surface in $S^6(1)$ in the case of $K \equiv 1/6$.*

Recently, Kozlowski and Simon [12] proved Corollary C by studying