Tôhoku Math. Journ.
38 (1986), 357-364.

ON A CLASS OF THE UNIVERSALLY INTEGRABLE RANDOM FUNCTIONS

TAKESHI SEKIGUCHI AND YASUNOBU SHIOTA

(Received June 19, 1985)

In the recent papers [4] and [5], Ogawa has developed the theory of a noncausal stochastic integral and proved that this integral naturally contains the symmetric integral of Stratnovich-Fisk as a special case. In this paper we give an extension of his result.

1. Introduction and preliminaries. First of all, we briefly review Ogawa's result. Let $\{W(x, \omega): 0 \leqq x \leqq 1\}$ be a one-dimensional Wiener process on a complete probability space (Ω, F, P), $\phi = \{\phi_n(x)\}$ be a complete orthonormal system (CONS, for short) in $L^2([0, 1])$ and set $K_n(x, y) = \sum_{k=1}^{n} \phi_k(x)\phi_k(y)$. In this section we assume that every random function is $B([0, 1]) \times F$-measurable and satisfies the condition $P\left(\int_0^1 F^2(x, \omega)dx < \infty\right) = 1$. We say that F is ϕ-integrable if

$$\lim_{n\to\infty} \int_0^1 F(x, \omega) \int_0^1 K_n(x, y)d^\circ W(y)dx$$

exists in probability and denote the limit by $\int_0^1 F(x)d_\phi W(x)$. Here $\int_0^1 K_n(x, y)d^\circ W(y)$ stands for the usual Ito (forward) integral. If the limit does not depend on the choice of a CONS ϕ, then we say that F is universally integrable and denote it by $^*\int_0^1 F(x)dW(x)$. Let $G(x, \omega)$ be a causal (i.e., adapted to the family of σ-fields $F_x = \sigma\{W(y): 0 \leqq y \leqq x\}$) random function and $H(x, \omega)$ be a (not necessarily causal) random function whose sample paths are of bounded variation with probability one. We say that the random function $F(x, \omega)$ of the form

$$F(x, \omega) = H(x, \omega) + \int_0^x G(y, \omega)d^\circ W(y) \tag{1}$$

is a quasi-martingale.

THEOREM (Ogawa [4] and [5]). (1) *Every quasi-martingale is ϕ-integrable if and only if the following condition is satisfied*:

$$\sup_n \int_0^1 u_n^2(x)dx < \infty . \tag{2}$$