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ON A RESULT OF K. MASUDA CONCERNING REACTION-DIFFUSION EQUATIONS

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Abstract. We give a simplified proof of a recent result due to K. Masuda concerning the global existence and asymptotic behavior of non-negative solutions to some reaction-diffusion systems. This new method also provides an analogous result under weaker growth assumptions on the nonlinear terms.

Introduction. Let Ω be an open, bounded domain of class C^1 in $\boldsymbol{R}^n$, with boundary $\Gamma = \partial\Omega$. Let d_1, d_2 be two positive constants with $d_1 \neq d_2$ and $\alpha_1(x)$, $\alpha_2(x)$ two nonnegative functions of class $C^1(\Gamma)$ with $\alpha_1 \leqq 1$, $\alpha_2 \leqq 1$. Let $\varphi \in C^1(\boldsymbol{R}^+)$ be a nonnegative function. We consider the reaction-diffusion system

$$(1)\qquad \begin{cases} \partial u/\partial t - d_1\Delta u + u\varphi(v) = 0 & \text{on} \quad \boldsymbol{R}^+\times\Omega \\ \partial v/\partial t - d_2\Delta v - u\varphi(v) = 0 & \text{on} \quad \boldsymbol{R}^+\times\Omega \end{cases}$$

with the homogeneous boundary conditions

$$(2)\qquad \begin{cases} \alpha_1\partial u/\partial n + (1-\alpha_1)u = 0 & \text{on} \quad \boldsymbol{R}^+\times\Gamma \\ \alpha_2\partial v/\partial n + (1-\alpha_2)v = 0 & \text{on} \quad \boldsymbol{R}^+\times\Gamma \end{cases}$$

A basic question, initially raised by Martin in [5], is the existence of global solutions in $C(\bar{\Omega})$ for the initial-value problem associated to the system (1)-(2). This question has been studied successively by Alikakos [1] who gave a positive answer when $\varphi(v) \leqq C(1 + |v|^{n+2/n})$, and by Masuda [6] who solved the question when $\varphi(v) \leqq C(1 + |v|^{\beta})$ with $\beta > 0$ arbitrarily large without any restriction on n.

In this paper, we show that the method of K. Masuda can be generalized to handle any non-linearity $\varphi(v)$ such that

$$(3)\qquad \lim_{v\to+\infty}[(1/v)\mathrm{Log}(1+\varphi(v))] = 0\,.$$

Also the proof given here is slightly simpler than Masuda's argument and is therefore still interesting when $\varphi(v) = cv^{\beta}$.

1. Notation and preliminary observations. Throughout the text we shall denote by $\|\ \ \|_p$ the norm in $L^p(\Omega)$, $\|\ \ \|_\infty$ the norm in $C(\bar{\Omega})$ or $L^\infty(\Omega)$