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INTERSECTION FORMULA FOR STIEFEL-WHITNEY HOMOLOGY CLASSES

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1. Introduction and the statement of results. The purpose of this paper is to study the relationship between the Stiefel-Whitney homology classes of mutually transverse Euler spaces and their intersection in an ambient PL-manifold. Besides manifolds, real analytic spaces are typical examples of mod 2 Euler spaces (cf. Sullivan [11]).

Let (A, B) be a pair of a polyhedron A and a subspace B of A such that rank $H_*(A, B; \boldsymbol{Z}) < \infty$. Denote by $e(A, B)$ the mod 2 Euler number of the pair (A, B). If $B \neq \emptyset$, we write $e(A) = e(A, \emptyset)$.

Let X be a locally compact n-dimensional polyhedron. The polyhedron X is said to be a mod 2 Eulder space (cf. [1], [5]), if the following hold for the subpolyhedron ∂X:

(1) ∂X is $(n-1)$-dimensional or empty.

(2) $e(X, X-x) = \begin{cases} 1 & (x \in X - \partial X) \\ 0 & (x \in \partial X) \end{cases}$

(3) if $\partial X \neq \emptyset$, then $e(\partial X, \partial X - x) = 1$ $(x \in \partial X)$.

Let K be a triangulation of a polyhedron X. Denote by K' the barycentric subdivision of K. If X is an n-dimensional mod 2 Euler space, the sum of all k-simplexes in K' is a mod 2 cycle and defines an element $s_k(X)$ in $H_k(X, \partial X; \boldsymbol{Z}_2)$, which is called the k-th Stiefel-Whitney homology class of X (cf. [1], [5]). We put $s_*(X) = s_0(X) + s_1(X) + \cdots + s_n(X)$. We define the mod 2 fundamental class in $H_n(X, \partial X; \boldsymbol{Z}_2)$ to be $[X] = s_n(X)$. If X is a $\boldsymbol{Z}_2$-homology manifold, then we know that $s_*(X) = [X] \cap w^*(X)$, where $w^*(X)$ is the Stiefel-Whitney cohomology class of X.

Let X be an n-dimensional polyhedron and let K be a triangulation of X. If the union of all n-simplexes are dense in X, the polyhedron is said to be pure n-dimensional. If X is a mod 2 Euler space of pure dimension PL-embedded in a PL-manifold M with $\partial X \subset \partial M$ and $X - \partial X \subset M - \partial M$, then X is called a proper PL-subspace in M. Let a and b be homology classes in $H_*(M, \partial M; \boldsymbol{Z}_2)$. We define the homological intersection by $a \cdot b = [M] \cap (([M] \cap)^{-1} a \cup ([M] \cap)^{-1} b)$.

The main result of this paper is the following:

THEOREM. *Let X and Y be* mod 2 *Euler spaces of pure dimension*