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ORTHOGONAL STANCE OF A MINIMAL SURFACE AGAINST ITS BOUNDING SURFACES

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0. Introduction. Suppose a minimal surface is spanning a certain nice surface besides the usual boundary curves. Then there arises a question: Under some additional condition, e.g., that the minimal surface considered is stable, does it touch the boundary surfaces orthogonally along the intersecting traces? Many soap film experiments such as those Gergonne observed, seem to have convinced us of the affirmative answer. On the other hand, mathematical descriptions suggesting these circumstances can also be found somewhere; indeed a mean orthogonal intersection in the weak sense is shown in Courant [1], pp. 207–208.

The present study is concerned with one of the simplest cases in this circle of ideas, namely, with an oriented minimal surface S of disk type, which spans partly a given Jordan arc γ, and whose remaining boundary arc complementary to γ lies on a sufficiently smooth surface T prescribed. Under these circumstances the minimal surface in question turns out to meet the base surface T orthogonally at almost all points of the intersection arc, which is our main assertion to be proved in the ultimate.

1. Preliminaries. 1.1. First of all we shall have to specify the base surface as well as the handle attached to it. Let G denote a simply connected region comprising the closed upper semi-disk $B=\{(u, v)\,|\,u^2+v^2\leq 1, v\geq 0\}$ in the (u, v)-plane. Consider a surface T parametrized by the C^1-mapping $T=T(u, v)$ of the (u, v)-plane into $\boldsymbol{R}^3$ with full rank 2 everywhere on G. Extremities of the handle γ to be settled on T are denoted by $\mathrm{P}_{\pm 1}$, which may have the respective coordinates $T(\pm 1, 0)$ without loss of generality.

In the following, the notation $|T|$, $|\gamma|$ shall mean the loci of the surface T and of the arc γ respectively, i.e., the bare point set free from any parametrizations as submanifolds.

1.2. For the sake of technical convenience we will require further nice properties on γ: Jordan simplicity, rectifiability and disjointness with $|T|$ except at both terminal points. Hence it admits representation as a continuous VB-function (Abbreviation for *function of bounded variation*) $\gamma=\gamma(\theta)$ on the semi-circle $\beta=\{e^{\sqrt{-1}\theta}\,|\,0\leq\theta\leq\pi\}$ in such a way that the point $\gamma(\theta)$ moves from P_{+1} to P_{-1} as θ increases from 0 until π and that $\theta_1\neq\theta_2$ implies $\gamma(\theta_1)\neq\gamma(\theta_2)$.