Tôhoku Math. J.
43 (1991), 75–101

FINITENESS OF A COHOMOLOGY ASSOCIATED WITH CERTAIN JACKSON INTEGRALS

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(Received December 20, 1989)

Abstract. A structure theorem on q-analogues of b-functions is stated. Basic properties for Jackson integrals of associated q-multiplicative functions are given. Finiteness of cohomology group attached to them is proved for arrangement of A-type root system. Some problems about the derived q-difference systems are posed. An example of basic hypergeometric functions are given.

1. Let $E_n := E^n$ be the direct product of n copies of an elliptic curve E of modulus $q=e^{2\pi\sqrt{-1}\tau}$ for $\mathrm{Im}\,\tau>0$. The first cohomology group $H^1(E_n, \boldsymbol{C})$ has the Hodge decomposition $H^1(E_n, \boldsymbol{C})=H^{1,0}(E_n)+H^{0,1}(E_n)$, where $H^{1,0}(E_n)$ is isomorphic to the direct sum of n copies of $H^{1,0}(E)$, the space of holomorphic 1-forms on E. Let $\{\mathfrak{z}, \cdots, \mathfrak{z}_n ; \mathfrak{z}_{n+1}, \cdots, \mathfrak{z}_{2n}\}$ be a basis of the first homology group $H_1(E_n, \boldsymbol{Z})$ such that each pair $\{\mathfrak{z}_j, \mathfrak{z}_{n+j}\}$ represents a pair of canonical loops in E. There exists a system of holomorphic 1-forms $\theta_1, \cdots, \theta_n$ on E_n such that

$$
\begin{aligned}
&\int_{\mathfrak{z}_j} \theta_k = 2\pi\sqrt{-1}\,\delta_{j,k} \\
&\int_{\mathfrak{z}_{n+j}} \theta_k = 2\pi\sqrt{-1}\,\tau\delta_{j,k}\,, \qquad \mathrm{Im}\quad \tau>0\,.
\end{aligned}
\tag{1.1}
$$

We denote by $\bar{X}$ the factor space of the dual $H^{1,0}(E_n)^*$ of $H^{1,0}(E_n)$ with respect to the abelian subgroup $A=\langle \mathfrak{z}_1, \cdots, \mathfrak{z}_n\rangle$ of $H_1(E_n, \boldsymbol{Z})$ generated by $\mathfrak{z}_j$, $1\leq j\leq n$. This is possible because $H_1(E_n, \boldsymbol{Z})$ can be contained in $H^{1,0}(E_n, \boldsymbol{C})^*$. In the same way we denote by X the factor space $H_1(E_n, \boldsymbol{Z})/A$. X can be assumed to be a submodule of $\bar{X}$ and has a basis $\chi_j=\mathfrak{z}_{n+j} \bmod A$. An arbitrary $\chi\in X$ is written uniquely as

$$
\chi=\sum_{j=1}^{n} \nu_j\chi_j \qquad \text{for} \quad \nu_j\in \boldsymbol{Z}\,. \tag{1.2}
$$

The quotient $\bar{X}/X$ is canonically isomorphic to E_n. By the map

$$
\bar{X}\ni\omega\mapsto x=(x_1=\exp((\theta_1, \omega)), \cdots, x_n=\exp((\theta_n, \omega)))\in(\boldsymbol{C}^*)^n \tag{1.3}
$$

for $\omega\in\bar{X}$, $\bar{X}$ is isomorphic to the algebraic torus $q^{\bar{X}}=(\boldsymbol{C}^*)^n$ and X is isomorphic to the discrete subgroup q^X generated by $q^{\chi_1}=(q, 1, \cdots, 1), \cdots, q^{\chi_n}=(1, 1, \cdots, q)$. Here (θ, ω) denotes the canonical bilinear form on $H^{1,0}(E_n, \boldsymbol{C})$ and its dual.

We denote by $R(\bar{X})$ the field of rational functions on $q^{\bar{X}}$ and by $R^{\times}(\bar{X})$ the