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A KIND OF ASYMPTOTIC EXPANSION USING PARTITIONS

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Abstract. It is shown that a certain algebraic identity involving a summation over partitions can be utilized to obtain a class of asymptotic expansions for large parameters. A number of special formulas related to some well-known number sequences and classical polynomials are presented as illustrative examples.

1. Introduction—Statement of problem. Throughout we will make use of formal power series (over the real or complex field) for which formal manipulations with ordinary addition, multiplication and substitution as well as formal differentiation are defined in the usual way. Logarithms and powers (with complex exponents) of a power series having a positive constant term are also defined formally. See Comtet's "Advanced Combinatorics" [4, §1.12, §3.5].

As usual, $\boldsymbol{Z}_+$ and $\boldsymbol{Z}_0$ denote respectively the sets of positive integers and of non-negative integers, and $\boldsymbol{C}$ the field of complex numbers.

Let $\phi(t)=\sum_{n\geqslant 0} a_n t^n$ be a formal power series with $a_0=\phi(0)=1$. Suppose that for any $\alpha\in\boldsymbol{C}$ with $\alpha\neq 0$ we have a formal power series expansion of the form

$$(\phi(t))^{\alpha}=\sum_{n\geqslant 0}\begin{Bmatrix}\alpha\\ n\end{Bmatrix}t^n\,,\qquad \begin{Bmatrix}\alpha\\ 0\end{Bmatrix}=1\,,\tag{1.1}$$

where the Taylor coefficient $\begin{Bmatrix}\alpha\\ n\end{Bmatrix}$, written in contrast with the notation for the ordinary binomial coefficient $\binom{\alpha}{n}$ just for convenience, may be determined by use of formal differentiation, namely

$$\begin{Bmatrix}\alpha\\ n\end{Bmatrix}=\frac{1}{n!}D_t^n(\phi(t))^{\alpha}\Big|_{t=0}\,,\qquad D\equiv d/dt\,.\tag{1.2}$$

Certainly, $\begin{Bmatrix}\alpha\\ n\end{Bmatrix}$ may also be expressed using the notation of Bell polynomials.

Let $\lambda\in\boldsymbol{C}$. Our main problem concerned is how to determine an asymptotic expansion of the function $\begin{Bmatrix}\lambda\alpha\\ n\end{Bmatrix}$ in terms of λ as $|\lambda|\to\infty$. As a number of well-known special

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