Tôhoku Math. J.
43 (1991), 149–160

NECESSARY AND SUFFICIENT CONDITIONS FOR "ZERO CROSSING" IN INTEGRODIFFERENTIAL EQUATIONS*

KONDALSAMY GOPALSAMY AND BIKKAR S. LALLI

(Received February 13, 1990)

Abstract. Necessary and sufficient conditions are obtained for all solutions of a class of linear scalar neutral-integrodifferential equations to have at least one zero. An application to an "equilibrium level-crossing" of a logistic integrodifferential equation with infinite continuously distributed delay is briefly discussed.

Introduction. There has been increased activity recently in the investigation of oscillatory nature of neutral delay differential equations. A prominent result obtained in these investigations is that a necessary and sufficient condition for the oscillation of all solutions of an autonomous neutral delay differential equation is that the associated characteristic equation has no real roots; there is a growing literature on this aspect (for example see [1], [5]–[9], [13]–[15]).

The purpose of this article is to derive a necessary and sufficient condition for all solutions of neutral integrodifferential equations of the form

$$\frac{d}{dt}[x(t)-cx(t-\tau)]+a\int_0^\infty K(s)x(t-s)ds=0\,;\quad t>0 \tag{1.1}$$

to have at least one zero on $(-\infty, \infty)$. Solutions of (1.1) which have at least one zero on $(-\infty, \infty)$ are said to have "zero crossings"; on the other hand if there is a solution x of (1.1) such that either $x(t)>0$ on $(-\infty, \infty)$ or $x(t)<0$ on $(-\infty, \infty)$, then such a solution is said to have no "zero crossings" (sometimes these solutions are said to stay away from zero). For literature related to stability characteristics of neutral integrodifferential equations we refer to Kolmanovskii and Nosov [12].

As an application of a special case of our result, we shall consider briefly "equilibrium level-crossing" of the solutions of the logistic integrodifferential equation

$$\frac{dN(t)}{dt}=rN(t)\left[1-\frac{1}{C}\int_0^\infty K(s)N(t-s)ds\right] \tag{1.2}$$

where

AMS subject classification 34K20, 92A15.

* Research supported by NSERC Grant A5293.