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ANOTHER PROOF OF THE DEFECT RELATION FOR MOVING TARGETS

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1. Introduction. The second main theorem and the defect relation of slow moving targets were discussed in [7], where Stoll gave the bound $n(n+1)$ for the sums of defects. The author generalized this result in [5] and gave in [6] examples of holomorphic mappings and moving targets which have the bound $n+1$. Ru and Stoll [3] then gave the bound $n+1$ in the general case. Since their proof is complicated, however, we give a simpler proof of Ru-Stoll's theorem in this paper.

2. Statement of the result. Let f be a holomorphic mapping of $\boldsymbol{C}$ into $P^n(\boldsymbol{C})$. Let $\tilde{f}=(f_0, \cdots, f_n)$ be its reduced representation, i.e., $\tilde{f}$ is a holomorphic mapping of $\boldsymbol{C}$ into $\boldsymbol{C}^{n+1}-\{0\}$. Fix $r_0>0$. We define the characteristic function $T(f; r)$ of f by

$$T(f; r)=\frac{1}{2\pi}\int_0^{2\pi}\log\|\tilde{f}(re^{i\theta})\|d\theta-\frac{1}{2\pi}\int_0^{2\pi}\log\|\tilde{f}(r_0e^{i\theta})\|d\theta$$

for $r>r_0$. In particular, the characteristic function of a meromorphic function is defined as that of the corresponding holomorphic mapping of $\boldsymbol{C}$ into $P^1(\boldsymbol{C})$.

For $q\geq n$, let g_j be $q+1$ holomorphic mappings of $\boldsymbol{C}$ into $P^n(\boldsymbol{C})$ with reduced representations $\tilde{g}_j=(g_{j0}, \cdots, g_{jn})$ $(0\leq j\leq q)$. Assume that the following conditions are satisfied:

(1) $T(g_j; r)=o(T(f; r))$ as $r\to\infty$ $(0\leq j\leq q)$;

(2) g_j $(0\leq j\leq q)$ are in general position, i.e., for any $j_0, \cdots, j_n$ with $0\leq j_0< \cdots < j_n\leq q$,

$$\det(g_{j_kl})_{0\leq k,l\leq n}\not\equiv 0 .$$

By (2), we may assume that $g_{j0}\not\equiv 0$ $(0\leq j\leq q)$ by changing the homogeneous coordinate system of $P^n(\boldsymbol{C})$ if necessary. Then put $\zeta_{jk}=g_{jk}/g_{j0}$ with $\zeta_{j0}\equiv 1$. Let $\mathfrak{K}$ be the smallest subfield containing $\{\zeta_{jk}\,|\,0\leq j\leq q, 0\leq k\leq n\}\cup\boldsymbol{C}$ of the meromorphic function field on $\boldsymbol{C}$. It is easy to check that $T(h; r)=o(T(f; r))$ as $r\to\infty$ for all $h\in\mathfrak{K}$. Furthermore, we assume

(3) f is non-degenerate over $\mathfrak{K}$, i.e., $f_0, \cdots, f_n$ are linearly independent over $\mathfrak{K}$. Put $h_j=g_{j0}f_0+ \cdots +g_{jn}f_n$. Then the counting function of g_j for f is defined by

$$N(f, g_j; r)=\frac{1}{2\pi}\int_0^{2\pi}\log|h_j(re^{i\theta})|d\theta-\frac{1}{2\pi}\int_0^{2\pi}\log|h_j(r_0e^{i\theta})|d\theta$$